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A WATER YIELD MODEL FOR SOLUTION OF TOTAL MONTHLY LOSSES WITHIN A WATERSHED

A THESIS

Presented to

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A WATER YIELD MODEL FOR SOLUTION OF

TOTAL MONTHLY LOSSES WITHIN A WATERSHED

Approved:

Chairman

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SUMMARY

The objective of this work was to formulate a mathematical model of monthly water yield from a watershed by least squares fitting techniques and, by use of this model, to generate individual monthly losses from monthly rainfall and streamflow data. Each loss term was obtained by subtraction of monthly precipitation excess, as calculated in the model, from monthly rainfall. This loss was, therefore, considered the summation of deep seepage and evapotranspirative losses.

The mathematical model consisted of two matrix algebra relations which were solved iteratively, to give a solution which converged. By making an initial estimate for the monthly precipitation excess, the time-distributive function of the watershed was computed. Using this recession function and the error from prediction of streamflow, the error in precipitation excess is found. This error is then applied to the initial estimate for precipitation excess and the whole procedure is repeated until the error term is minimal. Several restrictions were placed on the mathematics of the system in order for the model's output to be hydrologically sensible.

By use of a digital computer, the Burroughs B-5500, large data sets for several watersheds were processed to demonstrate the model's utility and aid in assessing its worth.

Results from the trial watersheds revealed a variety of recession curves which were interpreted to reflect the underlying geology of the region. Monthly loss values showed excellent relation

to the rainfall and geology of the respective regions. Individual loss values for the trial watersheds also showed proper congruity to the seasonal and rainfall variables of the respective areas.

CHAPTER I

INTRODUCTION

It is commonly recognized that precipitation of any form can be divided into two distinct components: that part which reaches a stream or watercourse, and that part which is lost to deep seepage and evapotranspiration. It is a recurrent problem to the hydrologist to ascertain what proportional amount of precipitation can be expected to occur as stream flow, or in other words, what the "yield" of a watershed will be. Any attempt, or model, designed to predict losses and precipitation excess has a multitude of variables which could be considered. Precipitation, antecedent precipitation, geology, intensity of precipitation, soil type, vegetation, evaporative losses, meteorological factors, and channel characteristics are but a few of the variables which can affect the balance of loss and runoff from a precipitation event.

Background

Various approaches to the problem of water yield or runoff prediction have been developed. One of the most basic is that of Linsley, Kohler, and Paulhus (1). Theirs is a graphical method of coaxial correlation presenting each separate storm as a runoff event. Other variables included are weeks of the year, duration of storm, an index of antecedent precipitation, and storm precipitation. By trial and error fitting of the original data points on a set of coaxial graphs, a predictor model for storm runoff can be constructed.

Benson (2) used drainage area, slope of channel, per cent of surface storage area, 24 hour rainfall, temperature, and an orographic factor in his flood prediction model which was solved by linear regression and correlation analysis. This model is easily adaptable to and highly recommended for use on digital computers. However, since this model has coefficients which lack analogous physical interpretation, the application of the model is limited to only prediction on the watersheds used to generate the equation.

Much research has also been done in the field of hydrograph analysis by Crawford and Linsley (3), Collins (4), Snyder (5), and others (6, 7, and 8). The models developed by Linsley and Crawford, known as the Stanford Watershed Models, utilize various increments of rainfall to generate a continuous outflow hydrograph by adjusting groundwater recession rates, evaporation losses, and infiltration capacities to the characteristics of a particular basin. Advocates of the Stanford Models believe that the coefficients obtained with this system can be used on a regional scale.

Collins derived a unit hydrograph from storm hydrographs by a method of successive approximation and residual error computations. A similar method suitable for use with digital computers was developed by Snyder, in which a least squares solution was used.

Snyder's method solved for the ordinates of the unit hydrograph, and then calculated the error between the observed and computed storm hydrographs. By regression of these errors and the ordinates of the unit hydrograph, values for loss estimates were obtained. To complete the iterative procedure, the estimates of losses were applied to the rainfall records to arrive at a better value for precipitation excess, that part of the rainfall which actually runs off without loss.

Snyder (9) later developed a model to predict monthly yield from a watershed by using monthly streamflow and rainfall records as input to generate numerical coefficients describing the time distribution of precipitation excess flow from a watershed. A sinusoidal loss parameter for seasonal variation was included, as was a time-trend variable. Loss terms from the model correlated well with pan evaporation readings, but were slightly out of phase. The model was designed for a base flow recession of 32 months duration with a damping system applied to the correction technique in order to suppress oscillation of the solution.

Objectives

Snyder's water yield model can generate an average loss term for each calendar month of the year, but cannot supply a continuous monthly loss or precipitation excess figure throughout the record. However, the methods described above for hydrograph analysis can synthesize an entire record, event by event. It is, then, the purpose of this work to create a model which combines the monthly time-base feature of a yield model and the capability of continuous synthesis in hydrograph analysis in order to generate a continuous monthly division of rainfall into precipitation excess and loss. It is further intended to relate the monthly loss, or anomalies in the loss patterns, to various characteristics of the watershed and to assorted meteorological phenomenon.

CHAPTER II

THE MATHEMATICAL MODEL

The water flowing in any watercourse is composed of two distinct parts of the excess precipitation: that which flows overland and that which flows more slowly underground. Therefore, from the moment the first particle of precipitation from a hydrologic event reaches the watercourse by an overland route until the last particle from that same event has migrated through the underlying soil to the same outflow point, there can be a considerable lapse of time. This means that streamflow for any time interval is composed of a certain percentage of the present time interval's precipitation excess plus a smaller percentage of the precipitation excess from the previous time interval plus another percentage of the precipitation excess from two time intervals past, and so on. In symbolic form, this relation can be represented as follows:

$$SF_{\theta} = O_1 PE_{\theta} + O_2 PE_{\theta-1} + O_3 PE_{\theta-2} + \cdot \cdot \cdot + O_{N+1} PE_{\theta-N} \qquad (1)$$

where SF is total streamflow during interval

O_1, O_2, ... O_N are time-defined percentages of excess

PE is precipitation excess which in turn is rainfall minus loss

θ is initial time interval

θ-N is time interval, N intervals previous

Assuming that the time interval to be used is one month and the percentage coefficients are constant from month to month throughout time, the simultaneous calculation of monthly streamflow volumes would appear as follows:

$$
\begin{aligned}
O_1 PE_N + O_2 PE_{N-1} + O_3 PE_{N-2} + \dots &= SF_N \\
O_1 PE_{N+1} + O_2 PE_N + O_3 PE_{N-1} + \dots &= SF_{N+1} \\
O_1 PE_{N+2} + O_2 PE_{N+1} + O_3 PE_N + \dots &= SF_{N+2} \\
O_1 PE_{N+3} + O_2 PE_{N+2} + O_3 PE_{N+1} + \dots &= SF_{N+3} \\
\dots \quad \dots \quad \dots \quad \dots &= \dots
\end{aligned}
\tag{2}
$$

Equations 2 can also be expressed as the product of a matrix and a vector as shown below:

$$
\begin{pmatrix}
PE_N & PE_{N-1} & PE_{N-2} & \dots \\
PE_{N+1} & PE_N & PE_{N-1} & \dots \\
PE_{N+2} & PE_{N+1} & PE_N & \dots \\
PE_{N+3} & PE_{N+2} & PE_{N+1} & \dots \\
\dots & \dots & \dots & \dots
\end{pmatrix}
\cdot
\begin{pmatrix}
O_1 \\ O_2 \\ O_3 \\ O_4 \\ \dots
\end{pmatrix}
=
\begin{pmatrix}
SF_N \\ SF_{N+1} \\ SF_{N+2} \\ SF_{N+3} \\ \dots
\end{pmatrix}
\tag{3}
$$

In compressed notation, Equation 3 becomes the following simple relation:

$$
(PE) \cdot \vec{O} = \vec{SF} \tag{4}
$$

Another method to accomplish the streamflow vector, SF, as found in Equation 3, is shown below:

$$\begin{array}{cccccccc} O_1 & & & & \cdots & PE_N & & SF_N \\ O_2 & O_1 & & & \cdots & PE_{N+1} & & SF_{N+1} \\ O_3 & O_2 & O_1 & & \cdots & PE_{N+2} & = & SF_{N+2} \\ O_4 & O_3 & O_2 & O_1 & \cdots & PE_{N+3} & & SF_{N+3} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots \end{array} \qquad (5)$$

This becomes, in similar compressed notation to Equation 4:

$$(O) \cdot \vec{PE} = \vec{SF} \qquad (6)$$

Equation 4 can be solved for the coefficients, (O), if monthly streamflow and precipitation records are available. Precipitation excess, PE, is available if a first estimate is made for watershed losses, L, because of the previously mentioned relation for the precipitation, RF:

$$RF = L + PE \qquad (7)$$

Once the O terms from Equation 4 have been solved, Equation 6 is used as a predictor for streamflow. If each "streamflow predicted" from Equation 6 is represented by $\widehat{SF}$, and it is assumed that the prediction cannot be absolutely correct, then the precipitation excess term must be composed of the true precipitation excess, PE, plus some error, E.

Therefore, Equation 6 becomes the expression:

$$(O) \cdot [\overrightarrow{PE + E}] = \hat{\vec{SF}} \qquad (8)$$

or, in expanded form:

$$(0) \cdot \vec{PE} + (0) \cdot \vec{E} = \vec{\widehat{SF}} \tag{9}$$

However, since (0) . PE is actually equal to SF, the observed streamflow, Equation 9 reduces to the following:

$$(0) \cdot \vec{E} = \vec{\widehat{SF}} - \vec{SF} \tag{10}$$

It can be seen in Equation 10 that (0) is known, SF is known, and $\widehat{SF}$ is computed through Equation 6 by using the original estimate of PE. Therefore, the error in the estimation of the precipitation excess, E, is the only unknown and can be solved through matrix operations. This error term is in turn used to correct the estimate of the PE in Equation 6 and the system becomes reduced to iterations of the above steps until the desired accuracy is attained.

The final forms for solution of Equations 6 and 10 become after development of the matrix algebra, the following:

$$0 = \left|(PE)^T(PE)\right|^{-1} \cdot (PE)^T \cdot \vec{SF} \tag{11}$$

$$\vec{\widehat{SF}} = (0) \cdot \vec{PE} \tag{12}$$

$$E = \left|(0)^T(0)\right|^{-1} \cdot (0)^T \cdot \left[\overrightarrow{\widehat{SF} - SF}\right] \tag{13}$$

where the exponents "T" and "-1" represent matrix transposition and inversion, respectively.

It is worthy to note that such a procedure in matrix algebra is simply a method of fitting by least squares and is a purely statistical technique. Figure 1 illustrates the flow of the mathematical processes within the model and, likewise, the flow of the computer operations in solving the model.

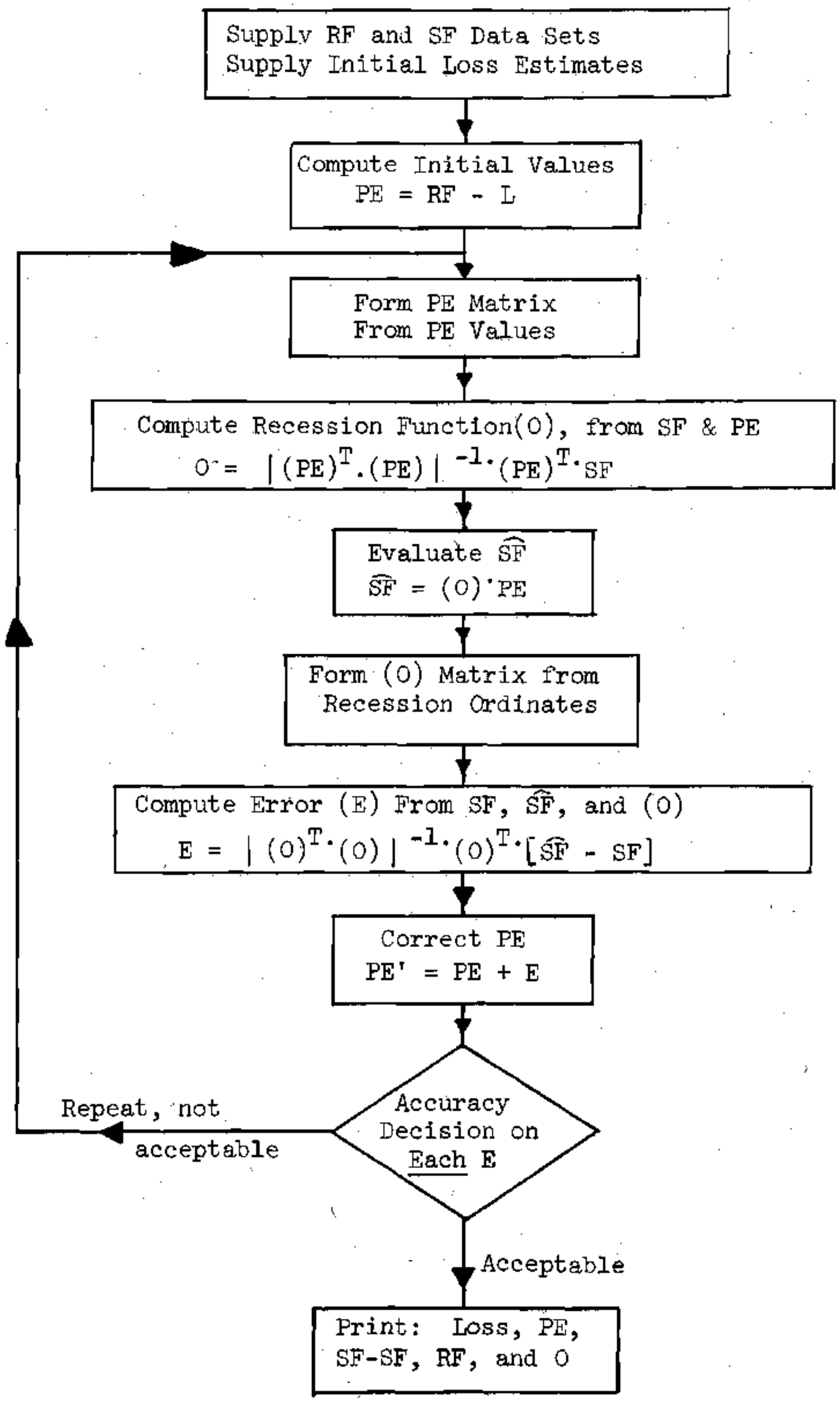


Figure 1. Flow of Mathematical Operations in Model

CHAPTER III

HYDROLOGIC AND PROGRAMMING CONSIDERATIONS

Following development of the mathematical model for water yield, it is necessary to provide a suitable working technique for application of the formulae. Several restrictions and limitations are governing factors in decisions regarding the construction of the model. A thorough understanding of the entire approach can best be understood by a review of the hydrologic, machine, and manpower limitations and restrictions that are imposed on the model.

Hydrologic Considerations

Recession Function Duration

As previously mentioned, the time necessary for all the precipitation excess from any one time interval to pass a gaging station often is substantial; in some cases, this period can be more than two years. This period, naturally, varies extensively from watershed to watershed; however, in construction of a model which will be used in any basin area, there must be an ample time allowance for a variety of base flow systems. The model under development in this research employs a 24-month recession function; that is, the precipitation 23 months previous to the month being analyzed is the last to contribute to that month's streamflow. Figure 2 shows a typical recession curve and the corresponding time scale of 24 months from the first to last monthly contribution, the ordinate being the value of the coefficient.

Unity Restriction

The recession function discussed is, in effect, a time distribution device for delayed runoff. Therefore, if the coefficients, which are percentages of the total monthly precipitation excess which becomes streamflow, are summed over the time of their distribution, the result must be unity. This continuity expression is stated as follows:

$$O_1 + O_2 + O_3 + \ldots + O_{24} = 1.0 \qquad (14)$$

This relation can then be incorporated in the solution for the coefficients, O, and eliminate one unknown by contributing one more equation to the system.

Limits on O Terms

Since the unknown values, O, represent the monthly percentage of PE that appears in the watercourse, it is unrealistic for these coefficients to be greater than one or less than zero. If these coefficients exceeded such limits, it would seem to imply that more PE was appearing than was initially generated, or that a negative amount of PE was flowing, respectively. Therefore, since there is no way to restrict the calculated O values obtained from Equation 11 during calculation, the computed values are scanned by the computer and any value greater than one is set equal to one, and any value less than zero is replaced by zero. In order to comply with the unity restriction, the correct values for O are scaled proportionally so that the total value of the summation of the O's remains equal to one.

Limits on Precipitation Excess

Precipitation Excess values for each month must have limitations

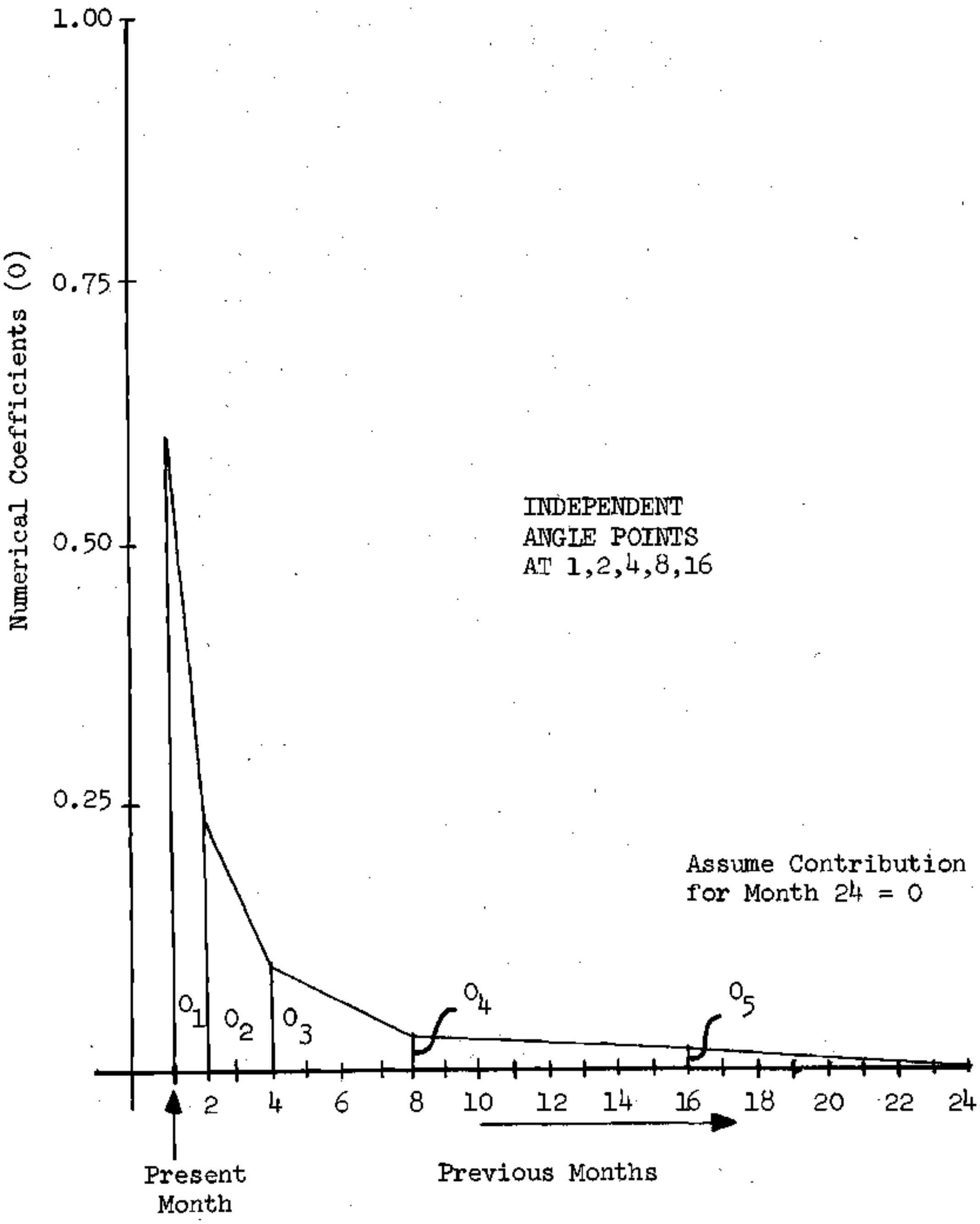


Figure 2. Typical 24-Month Recession Function

similar to the O coefficients. Negative precipitation excess is a physical impossibility; and there can never be more PE than there is precipitation, RF. Consequently, after the calculations for PE are made, the values are scanned and any value greater than the RF for that month is set equal to the RF and any value less than zero is set equal to zero.

Input Data Sets

In this model, it is imperative for all data sets to be continuous through time in order to insure proper functioning, since the streamflow, SF, for any time interval depends on the precipitation, RF, for each of the previous 24 months. Likewise, it is recognized by statisticians (10) that increasing the size of an experiment increases the accuracy of that experiment. This criterion makes larger data sets more valuable and increases the reliability of demonstrating the worth of the mathematical structure through data application. It must also be noted that a "lead-in" period of 23 months is necessary to supply the recession contributions to the PE for the first month calculated by the system. This implies that two years' data must be used to begin generation of loss terms; or, for example, if 15 years of monthly loss terms are desired as output, 17 years of data must be processed through the program.

Machine-Time Limitations

Recession Function Interpolation

By using a 24 month recession period, the solution for the coefficients, O, in Equation 11 contains 24 unknowns, or one for each contributing month. If some of these many ordinate values could be eliminated from the solution, the machine time required to compute the unknown coefficients would be greatly reduced. When operating on digital computers with matrix arrays, the time for computations varies exponentially with the size of such arrays, thus justifying any reduction in size of the matrices.

Snyder (11) has discussed in detail one method for eliminating unknowns which was shown suitable for hydrologic analyses. The technique discussed by Snyder, a method of linear interpolation of a continuous function, such as the water yield recession function, can be applied to provide a form-free fit of the respective function to any specified number of angle points within the function. It is necessary to have some knowledge of the function so that the most critical ordinates for proper function shape can be selected.

Figure 2 illustrates the type of linear segmented function that results from solving for a specified number of angle points within the continuous function. A typical recession function decays to such an extent that the ordinates toward the end of the recession can be well approximated by a straight line, while a sufficient number of ordinates must be specified toward the forepart of the recession in order to describe properly the rapid changes experienced by the function during that portion.

While the technique of linear approximation is useful for least squares fitting, it also has great utility for suppression of oscillations in solutions of this nature. Hydrologic functions have a tendency to oscillate when forced to zero, as does the tail of the recession function in this case; therefore, a linear approximation sometimes better describes the true hydrologic phenomenon than an irregular oscillation pattern which can occur by solving for many unnecessary ordinates.

For this computer program, it was decided to describe the straight line approximation of the recession function with independent angle point ordinates at one, two, four, eight, and sixteen months, respectively. The linear interpolation, then, yields ordinate values as follows:

$$
\begin{array}{lll}
o_1 = O_1 & o_7 = 1/4\ O_3 + 3/4\ O_4 & \cdots \\
o_2 = O_2 & o_8 = O_4 & o_{15} = 1/8\ O_4 + 7/8\ O_5 \\
o_3 = 1/2\ O_2 + 1/2\ O_3 & o_9 = 7/8\ O_4 + 1/8\ O_5 & o_{16} = O_5 \\
o_4 = O_3 & o_{10} = 3/4\ O_4 + 1/4\ O_5 & o_{17} = 7/8\ O_5 \\
o_5 = 3/4\ O_3 + 1/4\ O_4 & o_{11} = 5/8\ O_4 + 3/8\ O_5 & \cdots \\
o_6 = 1/2\ O_3 + 1/2\ O_4 & o_{12} = 1/2\ O_4 + 1/2\ O_5 & o_{24} = 0
\end{array}
\tag{15}
$$

Since the original 24 values of the recession ordinates have been reduced to only five, each fractional multiplication of precipitation excess, as seen in Equations 2, must be done in terms of those specific five as seen in Equation 15. Such an operation can be more simply understood by use of a numeric operator system as seen in Table 1. To further demonstrate how the operator system is employed, each row of the precipitation excess matrix used for solution of Equation 4 or Equation 11 should appear as seen below:

Column O_1	Column O_2	Column O_3	Column O_4	Column O_5
PE_{24}	PE_{23}	$1/2\ PE_{22}$	$1/4\ PE_{20}$	$1/8\ PE_{16}$
	$+1/2\ PE_{22}$	$+PE_{21}$	$+1/2\ PE_{19}$	$+1/4\ PE_{15}$
		$+3/4\ PE_{20}$	$+3/4\ PE_{18}$	$+ \cdots$
		$+1/2\ PE_{19}$	$+PE_{17}$	$+PE_9$
		$+1/4\ PE_{18}$	$+7/8\ PE_{16}$	$+7/8\ PE_8$
			$+ \cdots$	$+ \cdots$

where: PE_{24} is PE for 24th Month of Record

PE_1 is PE for first Month of Record, etc.

The structure shown above results from substitution of Equation 15 into Equation 2 and collecting terms with like O's. However, upon insertion of the continuity restriction of Equation 14, the number of unknown ordinates becomes one less than five, or four. The appearance of a PE matrix row then becomes the following:

Column 1	Column 2	Column 3	Column 4
PE_{23}	$1/2\ PE_{22}$	$1/4\ PE_{20}$	$1/8\ PE_{16}$
$+1/2\ PE_{22}$	$+\ PE_{21}$	$+1/2\ PE_{19}$	$+1/4\ PE_{15}$
$-3/2\ PE_{24}$	$+3/4\ PE_{20}$	$+3/4\ PE_{18}$	$+\ .\ .\ .$
	$+1/2\ PE_{19}$	$+\ PE_{17}$	$+\ PE_{9}$
	$+1/4\ PE_{18}$	$+7/8\ PE_{16}$	$+7/8\ PE_{8}$
	$3\ PE_{24}$	$+\ .\ .\ .$	$+\ .\ .\ .$
		$-\ 5\ PE_{24}$	$-\ 8\ PE_{24}$

The length of a column in the PE matrix, then, is the number of months of record minus the number of months required for a lead-in into the recession, 23, making the dimensions of the matrix (MO-23) by 4, where MO equals the total number of months in the data set.

Error Solution Interpolation

From Equations 11 and 13, it can be seen that both the solutions for the ordinates O and errors, E, utilize analogous equations. Therefore, a similar system of linear interpolation and numeric operators was applied to the O matrix and E vector of Equation 13 as was applied to Equation 11. In this case, the prediction error, E, is the variable which has an associated distribution function. For example, the total error in a predicted error term, $\widehat{SF}$-SF, is caused by the errors in precipitation excess from months past. The amount of influence from any one error should have a marked effect on the next month and decreasing influence on each month thereafter. Thus, each error of prediction has some dependence on preceding months, and, in turn, influences the prediction ahead of it in time. Realizing the possibility of over-solution to the problem by inference of too much information, the distribution of Figure 3 depicts the system used for this model. It is seen that the 24 month lead-in period has been retained; however, a three month error correction for months ahead in time has been added to the distribution to expedite a more rapid convergence of the system. From Figure 3, the independent ordinates are seen to be 11 in number, thus reducing the horizontal dimension of the O matrix from 27 to 11. A similar numeric operator technique to that shown in the previous discussion of the recession function w was used, and this system of operators is found in Table 3 in the Appendix, along with Table 4, the typical O matrix as it appears before linear interpolation has been accomplished. By limiting the size of the O matrix and sliding the $\widehat{SF}$-SF vector forward one month after each calculation, much computer time was saved; each month utilized the same O matrix repeatedly, instead of using a single one of a much larger size. Figure 4 shows the flow diagram of the sub-routine for computation of all the error terms. In this type of solution, it can be seen that the first 23 errors are not mathematically correct, but are only partially solved for the composite error. If solution is done rigidly for each month, another 2 years lead in period would be lost from the solution, but, in order to compromise, the partially-correct, first 23 errors were assumed

Table 1. Numeric Operator System For Recession Function

Month	Coefficients				
	O_1	O_2	O_3	O_4	O_5
1	1	0	0	0	0
2	0	1	0	0	0
3		1/2	1/2	0	0
4		0	1	0	0
5			3/4	1/4	0
6			1/2	1/2	0
7			1/4	3/4	0
8			0	1	0
9				7/8	1/8
10				3/4	1/4
11				5/8	3/8
12				1/2	1/2
13				3/8	5/8
14				1/4	3/4
15				1/8	7/8
16				0	1
17					7/8
18					3/4
19					5/8
20					1/2
21					3/8
22					1/4
23					1/8
24					0

acceptable. The forward influence of an error is terminated after three months so that two years phase-out time on the end of the record will not be lost.

The final form of the program used for computation is found in Appendix B. of this report. The program language used was Algol-Extended for use on the Burroughs B-5500 model computer. Time limitations were critical factors for program scope and matrix size.

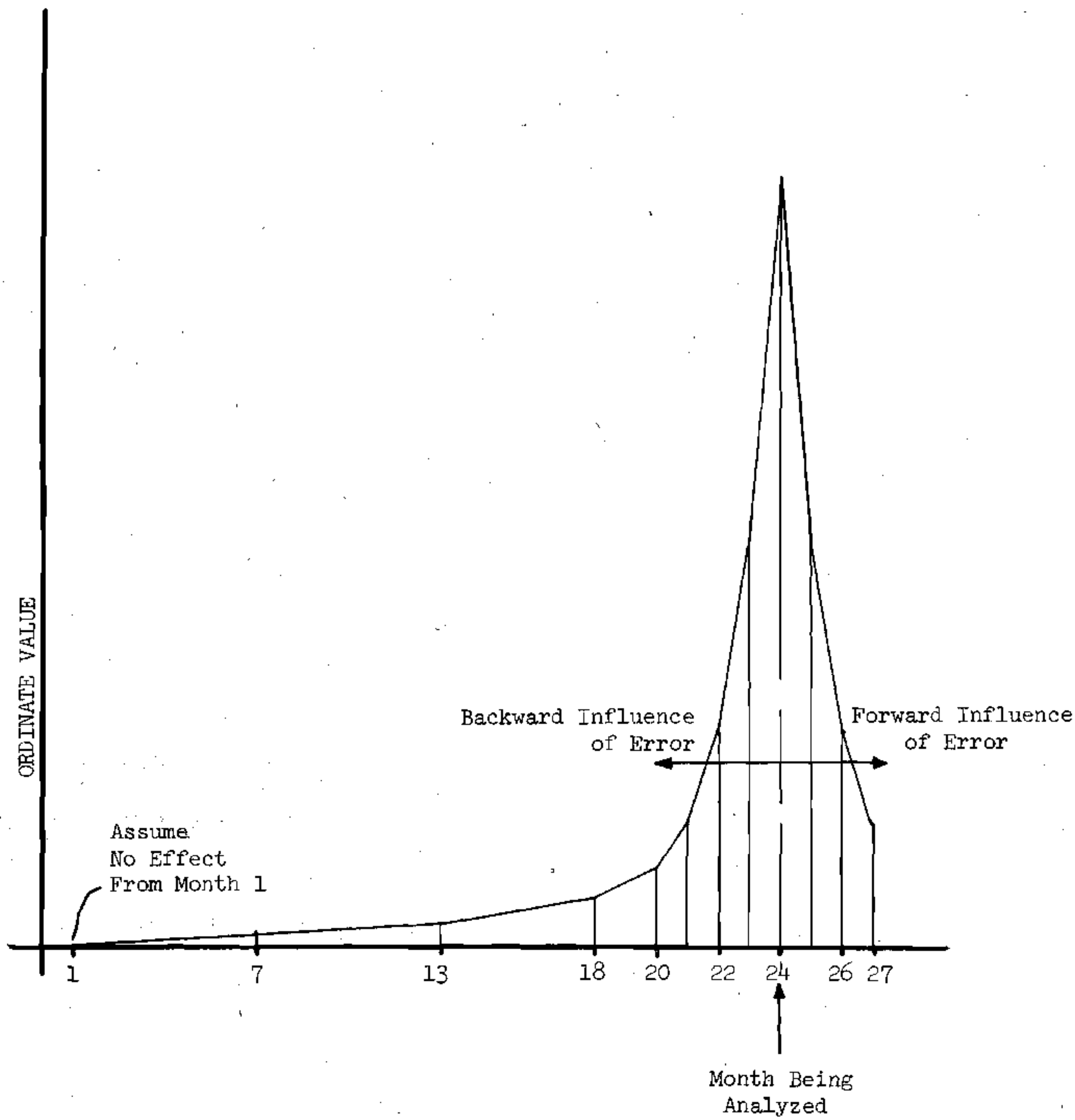


Figure 3. Typical Error Distribution Function

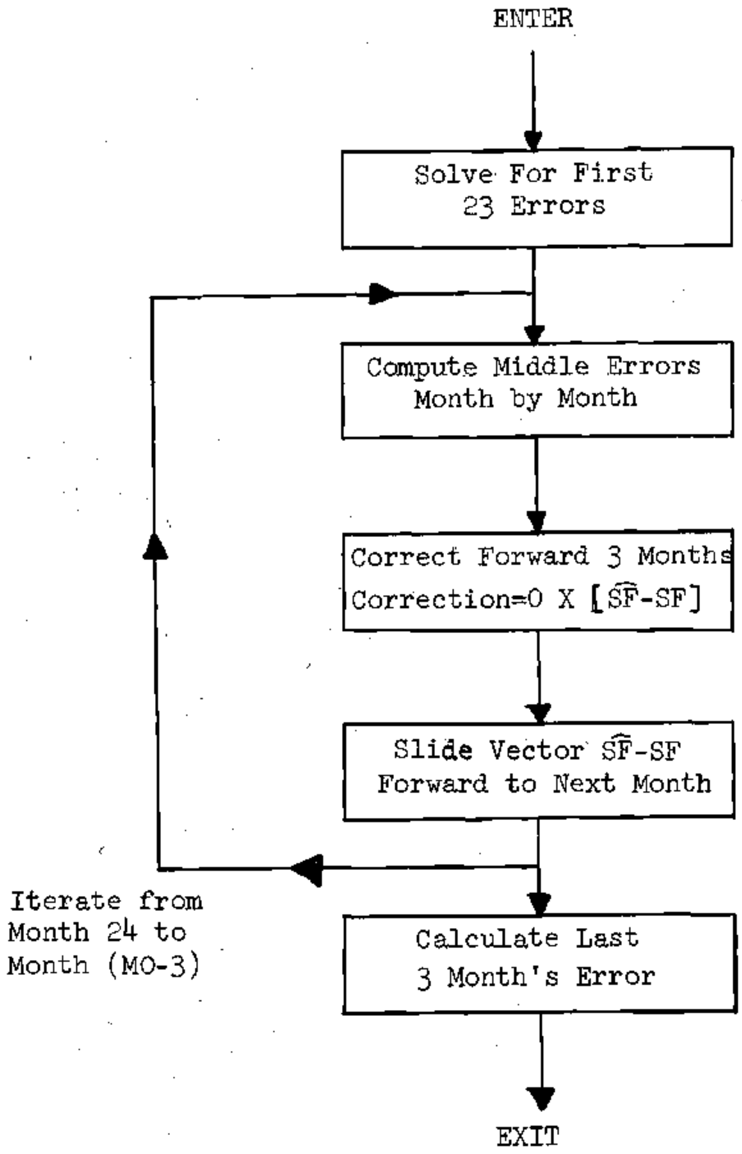


Figure 4. Flow Diagram for Error Calculations

CHAPTER IV

WATERSHEDS EMPLOYED

The Watersheds

Several watersheds were employed to test the ability of the model to compute monthly hydrologic losses from precipitation and streamflow data. Continuous data sets for a variety of regions within the continental United States were chosen from records available. Several Tennessee Valley Authority watershed studies were used and the remaining records were combinations of Weather Bureau precipitation records and United States Geological Survey streamflow data. Pertinent information about each watershed is discussed below.

White Hollow, Tennessee

White Hollow is a basin area of 1715 acres in Union County, Tennessee. Twenty-four years of monthly precipitation and streamflow records were available in a research study by the TVA on this watershed (12). The climate at White Hollow is temperate with an average annual precipitation of approximately 46 inches. The soils of the region are generally steep, cherty loams which are rated as fifth class by the U.S. Department of Agriculture rating scale. These type soils are the poorest for supporting agricultural growth and, consequently, the primary vegetation on White Hollow is a forest cover.

Bureau Creek, Illinois

This watershed of 101 square miles is an area in North-Central Illinois, upstream from a U.S. Geological Survey stream gaging station.

By combining U.S. Weather Bureau climatological data for precipitation at a station, known as Tiskilwa, with the streamflow records of the U.S.G.S., 24 years of record suitable for the model were obtained. Average annual precipitation is approximately 34 inches on the watershed. An agricultural environment exists on the glacial till formation which is typical of the North Central United States.

Pine Tree Branch, Tennessee

From another TVA hydrologic study (13), 20 years of record were available on an 88.2 acre watershed located in West Central Tennessee. The soil in this area is deep unconsolidated loess which is typical of the Eastern Mississippi Valley in the Central United States. Average precipitation on Pine Tree Branch is approximately 51 inches with the majority of this rainfall stemming from long-duration winter rains. The primary ground cover on this watershed is pine forests.

Parker Branch, North Carolina

Parker Branch Watershed is another of the TVA-sponsored hydrologic research projects (14), but only 10 years data were available. The drainage area contains 1060 acres situated just north of Ashville, in Western North Carolina. The soils of the region are generally loams and clay loams which are fairly erosive soils if they are not properly cultivated. Average annual precipitation on Parker Branch is approximately 38 inches with a monthly peak of rainfall occurring in the spring and early summer. Parker Branch contains a mixture of cover growth ranging from pine forests to agricultural crops.

Chestatee River, Georgia

This 153 square mile watershed is located in Northern Georgia and is one whose streamflow is measured periodically by the U.S. Geological

Survey. Nineteen years of data were obtained by correlating rainfall records at Dahlonega, Georgia and the published data of the U.S. Geological Survey. Yearly average precipitation within this watershed is approximately 61 inches. The soils of the region are generally clayey-limestones underlain by the metamorphic rock of the Blue Ridge mountains.

Big Coldwater Creek, Florida and Alabama

This 237 square mile watershed is located in Southern Alabama and the Florida Panhandle. Its elevation is less than 100 feet above sea level and it has a stable annual average precipitation pattern totalling approximately 65 inches. A 21 year data set was obtained from U.S. Geological Survey and Weather Bureau records. The soil of this region is a typical rich coastal sandy loam, and, although the area is near sea level, there are no marshes, but instead, pine forests.

Application Procedure

Data for each watershed was punched onto data processing cards, twelve months to a card, in chronological order, so that the computer could read all precipitation data, then streamflow data. All data sets initiated in January and terminated in December; the unit used for all data and computation was inches-per-month. In addition, one card was punched with 12 monthly average values for an initial estimate of losses. The computer was programmed to apply this 12 month average loss term to each respective monthly precipitation data term, in order to obtain an initial estimate for precipitation excess, PE. All sequential and iterative procedures in the program were controlled by a variable, designated MO, which represented the months of record in the data set being handled; this variable was compiled into each separate program by means of a statement setting MO equal to the months of record for the specific watershed.

As seen in Figure 1, the decision on the acceptability of results of calculation was the convergence of the error term, E. In general, the criterion programmed into the computer was that all values of E be less than 0.05 in order for the results to be accepted and thereby printed; however, for certain watersheds, which would not converge properly because of natural statistical oscillations or lack of machine time, either a definite number of iterations was specified or the acceptable value for E was changed accordingly.

Printed output was programmed to display the name of the watershed, the number of iterations required for solution, the precipitation and streamflow input records, the final monthly output loss terms, total losses during record (by month), ordinates of the recession function, monthly streamflow prediction error ($\widehat{SF}$-SF), and monthly residual error, E. A typical output format can be seen in Table 5 in the Appendix section.

CHAPTER V

INTERPRETATION OF RESULTS

Application Anomalies

In the process of applying the data of each individual watershed to the model through the computer program, several anomalies occurred which gave implications of minor procedural shortcomings within the framework of solution. Nonconvergence of the residual errors to near zero was the basic criterion for detecting occurrence of these anomalies, and almost all of the problems associated with the solution resulted in the aforementioned lack of convergence in this model.

First, it was found that the length of record for the data set being processed was related to the stability of the solution process. For example, Parker Branch, with a 10 year record, quickly approached the accuracy limit of 0.05 for each E term but the streamflow prediction error, $(\widehat{SF}\text{-}SF)$, and the recession function were unstable even after 50 iterations. On the other hand, there appeared to be an upper limit on the length of a data set for solution within a reasonable time on the electronic computer. Pine Tree Branch, 20 years record; White Hollow, 24 years record; and Bureau Creek, 24 years record, were all solved within a reasonable time limit, while Big Coldwater Creek, 21 years record, and Chestatee River, 19 years record, showed no signs of convergence after even 35 and 40 iterations. Although statistical analyses of input data was not undertaken, extremes in variability of precipitation events on certain watersheds could have been a cause for solution or lack of solution on those watersheds. It appeared by simple scanning of the data, that those watersheds with very irregular rainfall patterns and extreme variability were solved, and those whose monthly precipitation patterns showed more consistency were unsolved. Disregarding speculation about the cause, the effect of such nonsolution on some watersheds is to cause the limit on computer time to be exceeded. In iterative procedures such as those employed by this model, the time required for solution can sometimes be relatively large, and since this investigation was carried on under a time limit of 15 minute process time and one minute input-output (IO) time on the computer, this limitation became an obstacle to proper solution.

Another anomaly detected was the fact that solution was severely hindered by the forward correction technique, discussed in connection with Figure 4, or conversely, that solution proceeded much more rapidly with this mathematical operation omitted. Again, with no conclusive method to isolate the cause, it appears that during the first few iterations, the residual error, E, is high for most months, and if these errors were carried forward during the period when they were extremely large, the tendency would be for all E terms to remain large, or possibly become larger, thus causing a diverging or stagnant solution. Possibly, the ideal method for implementation of the technique would be to omit the forward correction until the solution was fairly stabilized, after perhaps five to ten iterations, then insert the correction procedure with the intent of forcing a more rapid solution. Such a maneuver is not practical unless one is able to operate either the program or the computer upon his discretion as to when each technique should or should not be employed.

Various attempts were made to speed up solution of the program, but on Big Coldwater Creek and Chestatee River, the efforts were to no avail. The first attempt was to substitute monthly coefficients of precipitation excess for multiplication by respective monthly precipitation values instead of subtracting an average value of monthly losses based on evaporation data. No significant change was noticeable with such a change, so an attempt was made to place a damper on the large changes which resulted in the variables during the first part of the solution. It was believed that if only a certain percentage of a calculated correction value was applied, the solution should stabilize more rapidly by decreasing the amount of change and magnitude of oscillation. The streamflow prediction error, $(\widehat{SF}-SF)$, was the variable used in this effort. After each $(\widehat{SF}-SF)$ term was computed, it was multiplied by 0.50 and the resulting values were used in calculation of E, as seen previously in Equation 13. The solution might possibly be expected to take longer, but more stability should have been obtained. However, upon application of this technique to the Big Coldwater Creek and Chestatee River watersheds, no significant change was discernable.

It is quite possible that these questions could be answered by exploratory analysis of additional sets of data, or by additional modifications to the numeric iteration technique to achieve non-oscillatory solutions. However, such exhaustive analysis is beyond the time limitations of this study.

Recession Function

The shape of the recession function which was found by solution of each respective data set offers a feedback system to check to see if proper adjustments are made for size and physical characteristics of the watershed. To be more specific, a function which decays slowly would seem to indicate a large watershed, or could signify the possibility of an underlying geologic formation which is conducive to ground water storage and flow. In a similar fashion, a recession curve which has large ordinate values toward the end of the recession tail or other unphysical properties has probably not been solved to the extent for which the model was designed. Table 2 shows the independent angle points calculated for each watershed and the accuracy or number of iterations under which they were run. Figure 5 illustrates the entire recession function for Pine Tree Branch and Bureau Creek, two of the more stable solutions of the six watersheds employed. It can be seen for the Bureau Creek watershed, which is located in a glacial till geological area, that over 95 percent of the precipitation excess from any month's precipitation occurred during that month, while in Pine Tree Branch, which is located on deep unconsolidated soil, the portion of precipitation excess calculated during the first month was only about 67 percent of the monthly rainfall. These results are physically sensible; in a glacial soil, there is little deep seepage, while in sandy loess formations, very much infiltration takes place which, in turn, slows down the runoff process. Minor oscillations can be seen in the tails of the recession functions, but no technique to eliminate these small fluctuations was found. These oscillations are probably the result of statistical operations and are of such a small scale as to not warrant undue concern.

Any attempt to interpret size of the watersheds found in this report from the recession function would be futile because each watershed

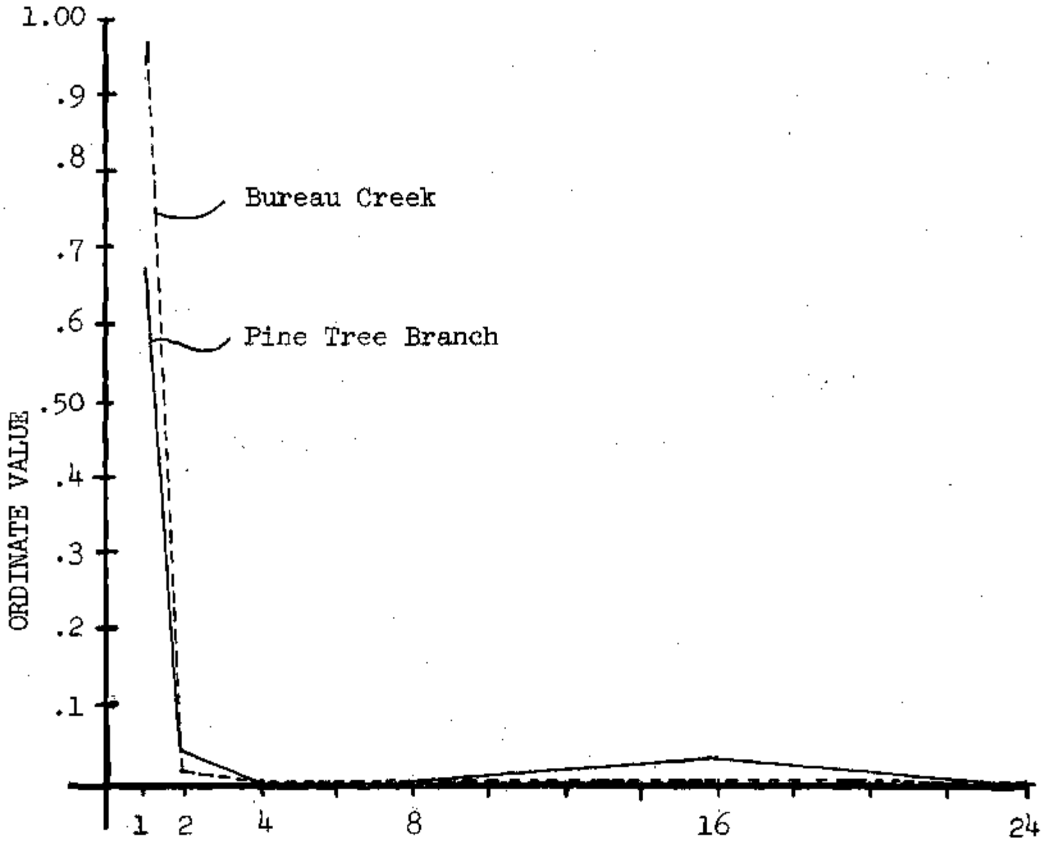


Figure 5. Recession Function for Bureau Creek and Pine Tree Branch Watersheds

Table 2. Recession Ordinates Calculated by Model

Watershed	O_1	O_2	O_4	O_8	O_{16}	Solution Criteria
White Hollow	.624	.047	.000	.000	.039	E < 0.05
Bureau Creek	.966	.013	.000	.004	.000	E < 0.05
Pine Tree Branch	.674	.029	.000	.001	.035	E < 0.05
Parker Branch	.296	.058	.016	.000	.083	E < 0.02
	.287	.051	.022	.000	.085	50 Iterations
Chestatee River	.334	.311	.000	.015	.015	25 Iterations
Big Coldwater Creek	.139	.124	.016	.066	.037	12 Iterations
	.131	.068	.082	.033	.040	35 Iterations

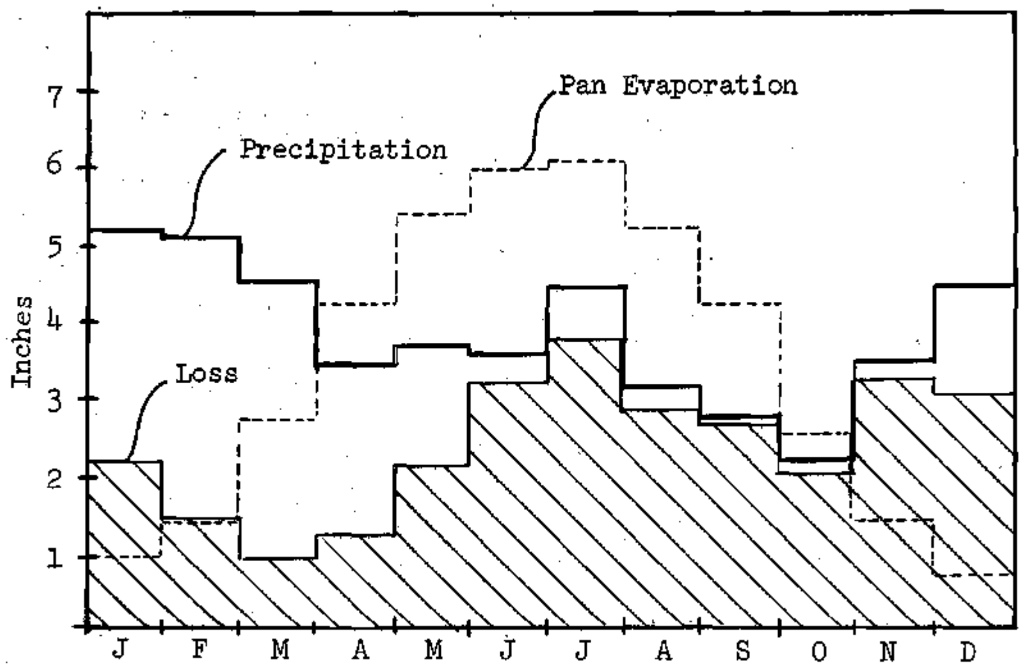
used was considered small enough so that only one precipitation measuring station could be used for correlation with streamflow data. If networks of precipitation stations were established over larger, possibly homogeneous areas, the shape of the recession function might be used for size comparison.

Those watersheds whose recession functions have improper shape are those which were not accurately and completely solved as a result of one of the reasons found in the preceding section of this chapter.

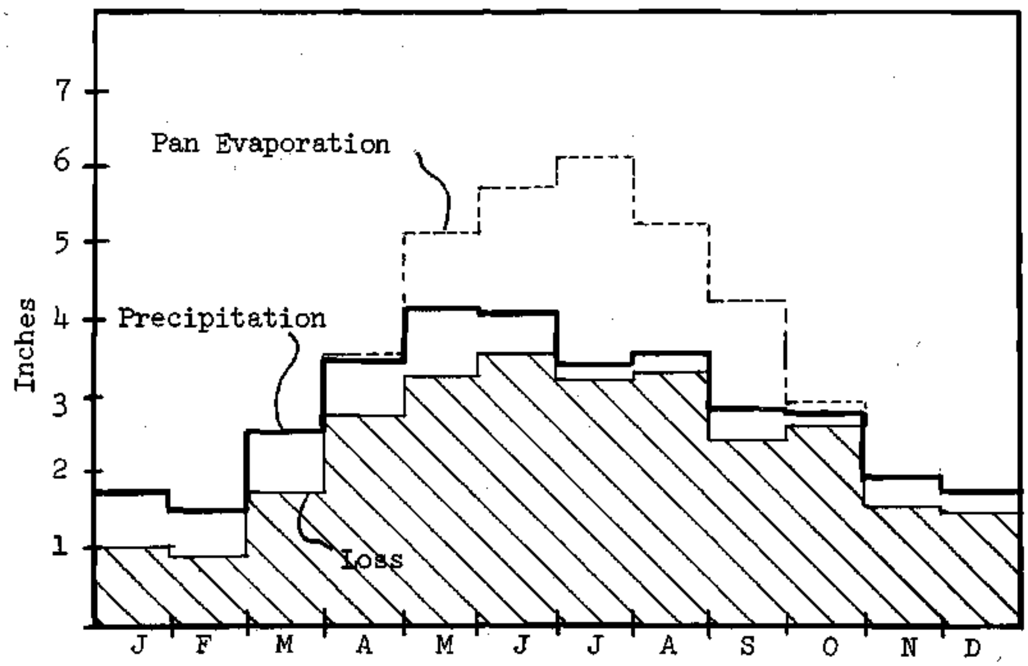
Average Loss-Rainfall Relationships

One of the most interesting features of the results obtained by application of the watersheds to the model was the relation between average monthly rainfall and computed monthly losses. Figure 6 and Figure 7 are the graphical expressions of relation between the two with the added features of monthly average pan evaporation in the vicinity of each watershed. There are several hydrologic facts which can be examined in order to discern the worth of the results depicted in Figures 6 and 7. First, monthly loss can never be greater than monthly rainfall. Second, when losses are greater than potential pan evaporation, the difference between the two must be either loss to deep seepage, or replenishment of soil moisture, or both. Lastly, it can be seen that monthly total precipitation excess is actually the difference between monthly rainfall and monthly loss. Thus, some idea of the wetness condition of a watershed during the year should be revealed by the variation of this parameter throughout the year.

Each of the above statements can be shown to exist in the four watersheds which were solved satisfactorily. Analytical results for



White Hollow



Bureau Creek

Figure 6. Average Loss-Rainfall Relationships for White Hollow and Bureau Creek Watersheds

these areas are shown in Figures 6 and 7. The others, Big Coldwater Creek and Chestatee River, did not reach a final solution, so average losses would be meaningless. With regard to the first criterion, each average monthly loss term can be seen to be less than each average monthly precipitation value for every month on every watershed. As to the second hydrologic statement discussed above, compliance by the model can be shown in several instances. During November and December at White Hollow, losses are substantially greater than pan evaporation. This phenomenon is very logical because after the summer months, the soil moisture profile is depleted below field capacity for the soil, and when the evaporation drops below the monthly rainfall, the first loss detected should go to replenishment of the soil moisture up to field capacity. This natural reaction is not detected on the Bureau Creek watershed because all three variables are in phase with one another, as seen in Figure 6. However, Pine Tree Branch, which is situated on deep unconsolidated soils, displays another characteristic of the second hydrologic criterion. This watershed, with its deep permeable substratum, seems to have approximately the same amount of losses each month, regardless of the season. This leads one to believe that there is substantial loss to deep seepage because for six months (October through March), losses are greater than or approximately equal to, potential pan evaporation, while the losses remain almost constant. Once the soil moisture is restored to field capacity, the loss during the late winter (January through March) is most likely penetrating deep within the soil layer.

The soil wetness parameter, or available moisture for runoff, is also well demonstrated by the fact that during the growing season when

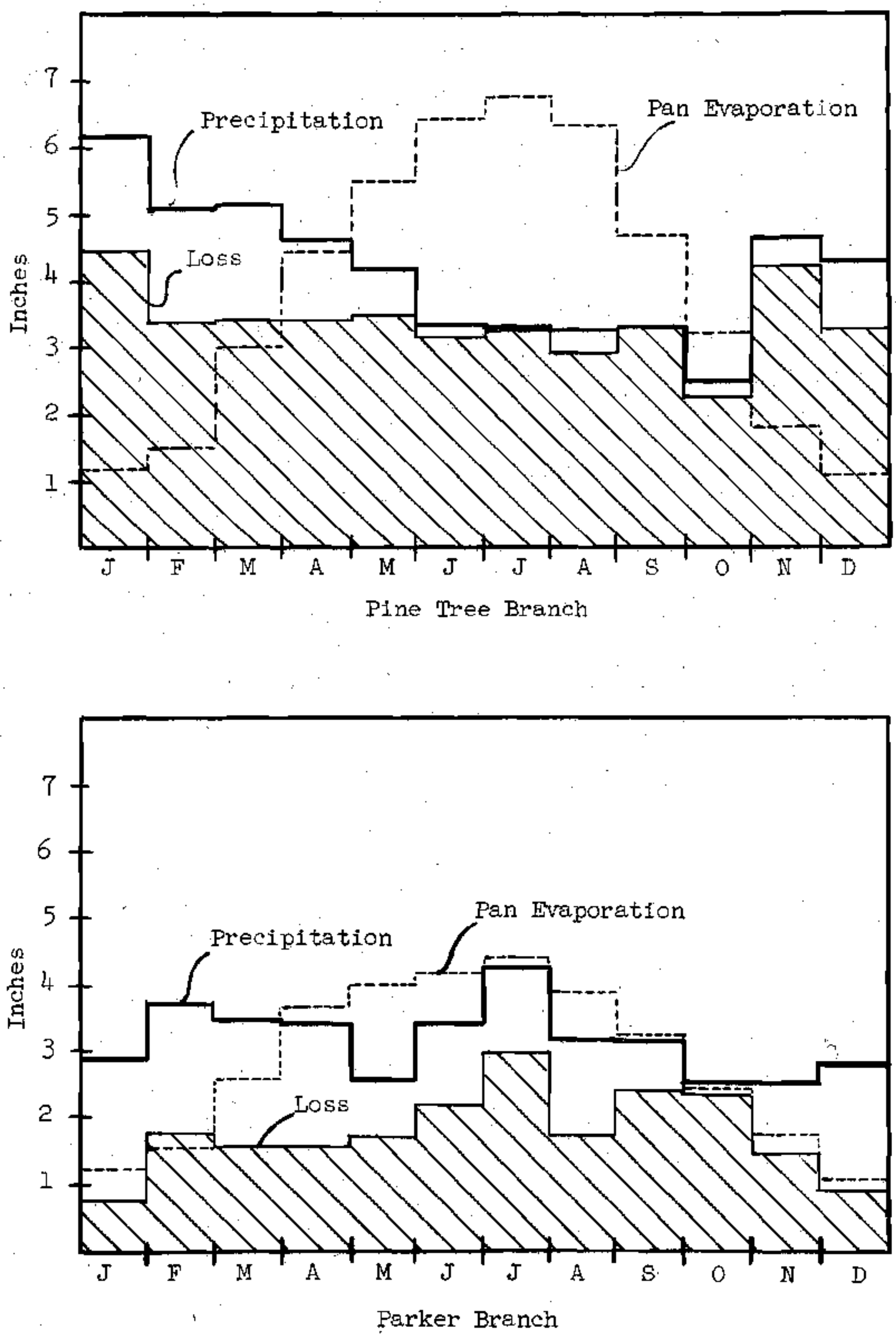


Figure 7. Average Loss-Rainfall Relationships for Pine Tree Branch and Parker Branch Watersheds

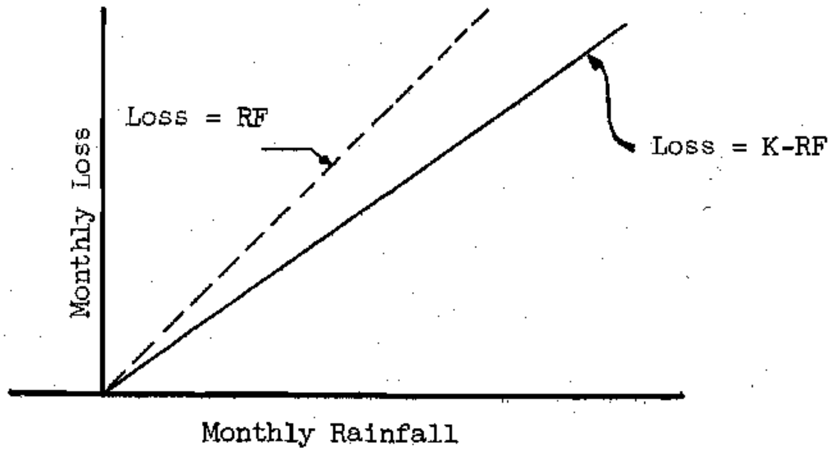
there are large losses to evapotranspiration, there is very little precipitation excess. The converse is also shown; during the winter months when plant growth is minimal, the precipitation excess is at a maximum, especially after the depleted soil moisture is restored. Each watershed in Figures 6 and 7 display evidence to this effect. Thus, it can be seen that, on an average monthly basis, the model actually produced hydrologic information which was indeed typical of the meteorological and geological characteristics of the region.

Individual Loss-Rainfall Relationships

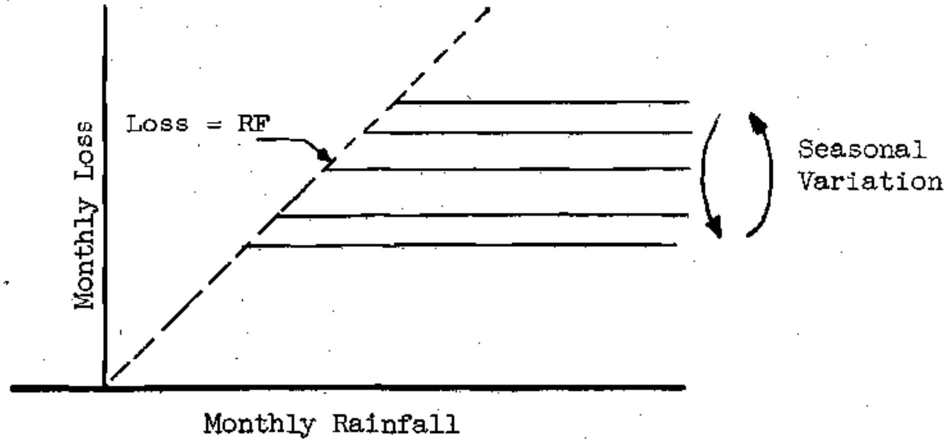
Regardless of what has been demonstrated to this point, the individual monthly loss term is the unknown value which this study originally set out to find. It is probably true that the hydrologically reasonable recession functions and average monthly losses which have been shown to exist through solution of the model were the result of proper solutions for individual monthly loss terms, but the monthly loss terms themselves must also be demonstrated to have hydrologic meaning.

The loss term for any month can be related to two general groups of variables: the seasonal, or climatic variables, and the physical variables, such as geological structure, which are inherent in the watershed. In every watershed, naturally, there is some interaction between these two variables and each loss term depends both on the seasonal and the geological mechanism. Seasonal variation can easily be represented by monthly progression through the year, but geologic variables are not as readily quantified. Since the geologic formations and physical characteristics of a watershed have not been represented by numerical values, another variable must be found which directly reflects the geology of an area. Rainfall was found to be a parameter which did just this; for example, if there is no seasonal or climatic effect to be considered, then loss is proportional only to rainfall through the distribution characteristics, or geology, of the region. This situation is depicted in Figure 8a. Conversely, if the geology of a region is ignored and only seasonal characteristics are considered, the loss should be independent of the precipitation as seen in Figure 8b. A more realistic combination of the two is shown in Figure 8c.

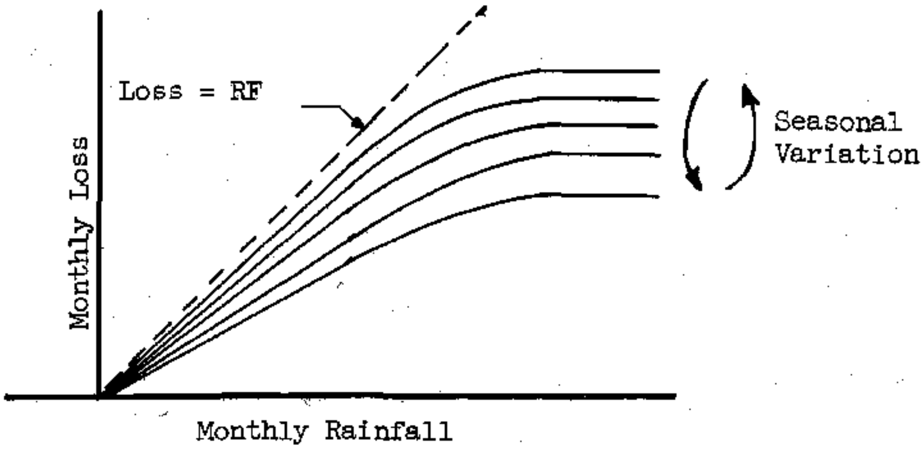
Individual monthly rainfall and loss values for White Hollow, Bureau Creek, and Pine Tree Branch watersheds were plotted and curves for each month were drawn as shown in Figures 9, 10, and 11, respectively. These best fitting lines showed that a distinct seasonal cycle existed. It is noteworthy that the amount of spread of the twelve monthly curves shows the tendency toward rainfall-geology dependence (Figure 8b) and climatic-seasonal dependence (Figure 9a). White Hollow, Figure 9, for example, shows many more seasonal tendencies than does Pine Tree Branch (figure 11). This is, indeed, a physical truth. Pine Tree Branch, as previously discussed, loses water from the bottom of its deep soil profile in proportion to the incoming rainfall, while White Hollow has a very erosive, impermeable soil making the seasonal parameter much more important in affecting losses. Bureau Creek, (figure 10), is a combination between the two other watersheds which indicates an interaction between rainfall and season. Maximum and minimum curves on the cyclical variations are indicative of true loss patterns, which are minimal in late winter and maximum in late summer. There is also some tendency for the losses to become independent of precipitation when the precipitation is high as shown by the curves becoming parallel to the rainfall axis.



8a. Loss Depends on RF Only



8b. Loss Depends on Season Only



8c. Loss Depends on Both Rainfall and Season

Figure 8. Theoretical Dependence of Losses on Rainfall and Seasonal Parameters

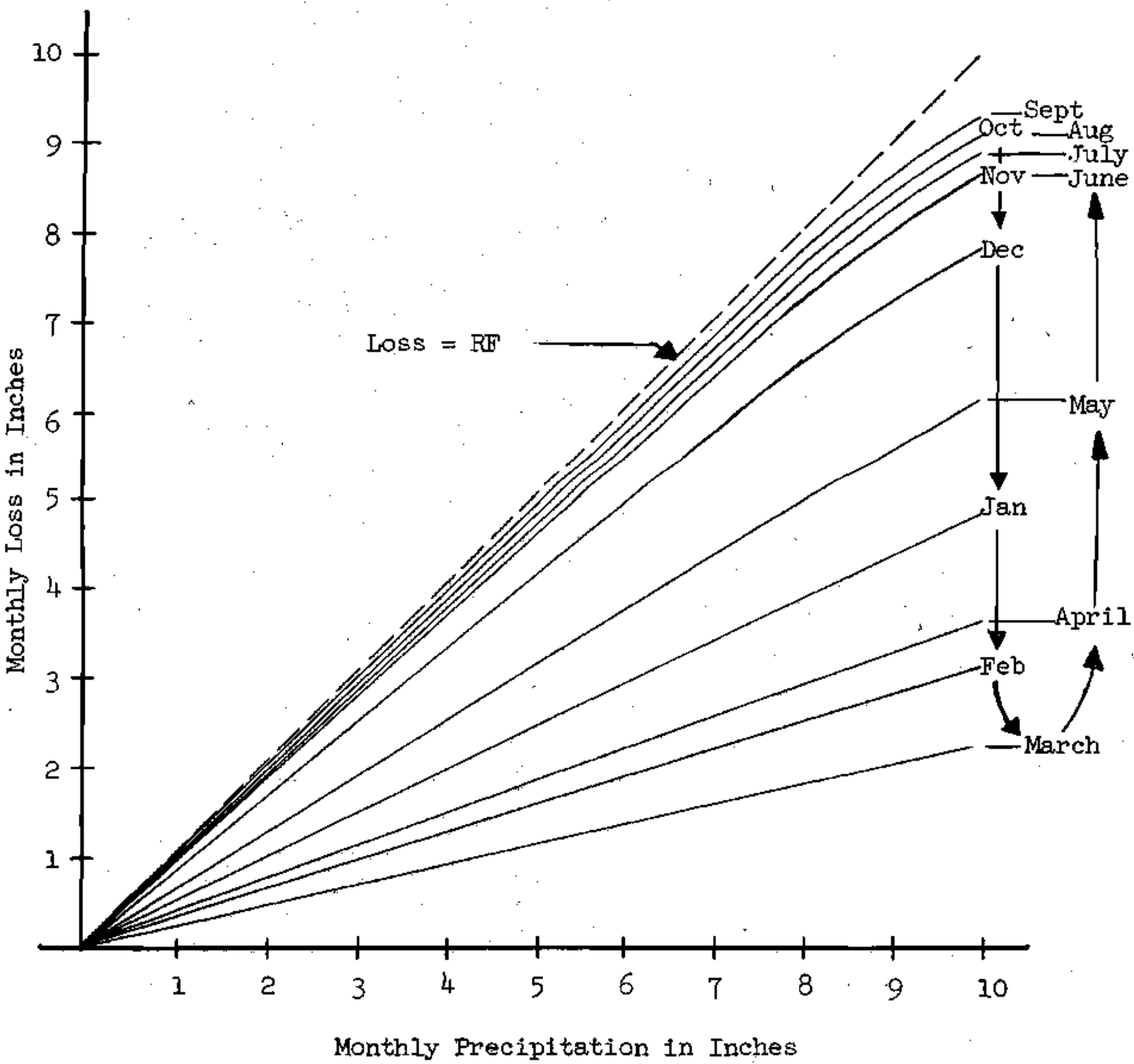


Figure 9. Individual Loss-Rainfall-Seasonal Relationships for Watershed White Hollow

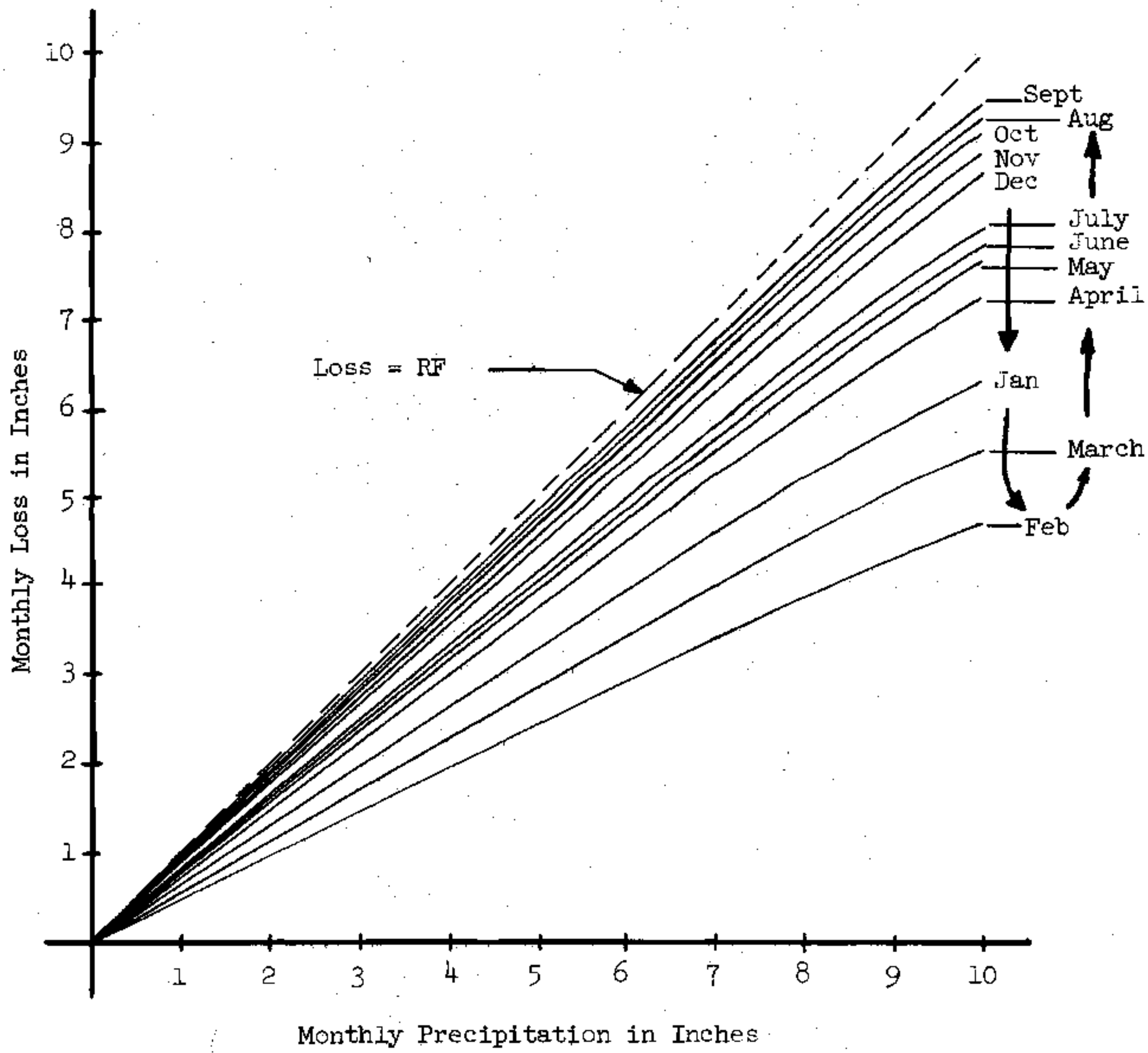


Figure 10. Individual Loss-Rainfall-Seasonal Relationships for Bureau Week Watershed

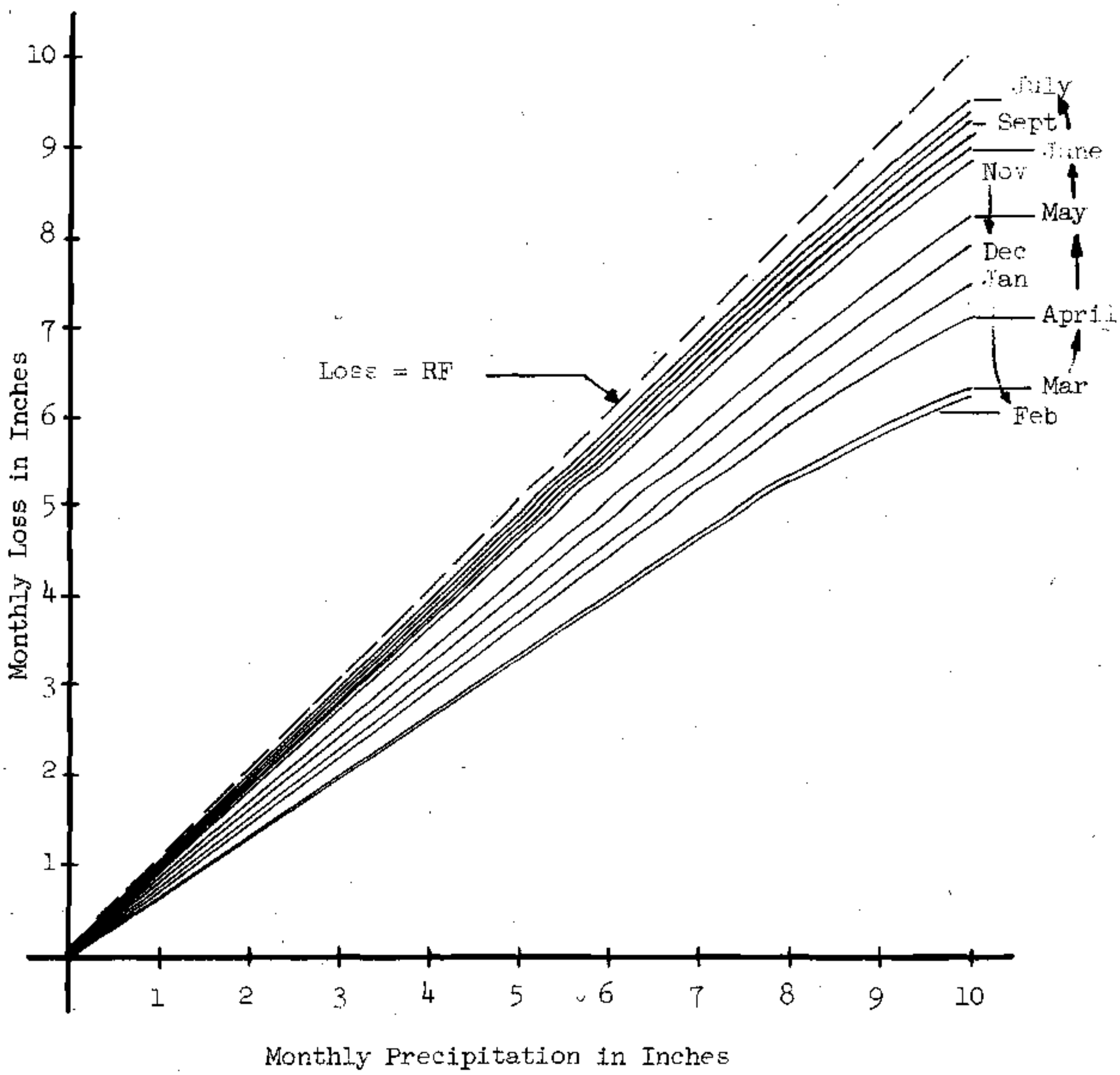


Figure 11. Individual Loss-Rainfall-Seasonal Relationships for Pine Tree Branch Watershed

CHAPTER VI

CONCLUSIONS

The model constructed in this report performed as postulated; individual monthly losses were calculated from streamflow and precipitation data. These results are a unique new form of hydrologic information which has a bright future for development of accuracy and reliability. Some of the more specific findings are listed below:

1. With ample computer time and proper restrictions, the computer program can generate monthly loss terms from monthly streamflow and precipitation data.

2. Average monthly loss terms calculated by the model are [illegible] hydrologically sensible.

3. Individual monthly loss terms calculated by the model can be shown to be very reasonable when related to seasonal and rainfall parameters.

4. Recession Functions found through the study were consistent with the known geology and soils of the watersheds.

5. Length of record affects the ability for solution on both extremes. Short records do not get proper statistical averaging and lengthy records utilize too much machine time.

6. The forward correction technique did not aid in more rapid solution when utilized throughout the duration of iterations in a computation.

CHAPTER VII

RECOMMENDATIONS

The limited time and resources for this research precluded many refinements and further study of the constructed model. The following suggestions are made as worthy revisions and future study of the model:

1. A shortening of the recession function duration from two years to six or nine months.

2. A shortening of the hydrologic interval of data collection from one month to one week or even one day.

3. The usage of networks of rainfall stations with larger watershed areas.

4. The correlation of individual losses computed to solar radiation or pan evaporation data in order to show better the dependence of losses upon meteorological variables.

5. Development of a method to ascertain the tendency toward convergence during solution.

6. Implementation of standard techniques to force convergence if the solution is ascertained to have no statistical tendency for convergence.

APPENDIX A.

Table 3. Numeric Operators for Error Distribution Function

Month	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}
1	0										
2	1/6										
3	1/3										
4	1/2										
5	2/3										
6	5/6										
7	1										
8	5/6	1/6									
9	2/3	1/3									
10	1/2	1/2									
11	1/3	2/3									
12	1/6	5/6									
13	0	1									
14	0	4/5	1/5								
15	0	3/5	2/5								
16	0	2/5	3/5								
17	0	1/5	4/5								
18	0	0	1								
19	0	0	1/2	1/2							
20	0	0	0	1							
21	0	0	0	0	1						
22	0	0	0	0	0	1					
23	0	0	0	0	0	0	1				
24	0	0	0	0	0	0	0	1			
25	0	0	0	0	0	0	0	0	1		
26	0	0	0	0	0	0	0	0	0	1	
27	0	0	0	0	0	0	0	0	0	0	1

Table 4. Typical O Matrix Before Interpolation

o_1	0	0	0	0	0	0	...	0	0	0	0
o_2	o_1	0	0	0	0	0	...	0	0	0	0
o_3	o_2	o_1	0	0	0	0	...	0	0	0	0
o_4	o_3	o_2	o_1	0	0	0	...	...	...	...	...
o_5	o_4	o_3	o_2	o_1	0	0	...				
o_6	o_5	o_4	o_3	o_2	o_1	0	...				
o_7	o_6	o_5	o_4	o_3	o_2	o_1	...				
o_8	o_7	o_6	o_5	o_4	o_3	...					
o_9	o_8	o_7	o_6	o_5	...						
o_{10}	o_9	o_8	o_7	...							
o_{11}	o_{10}	o_9	...								
o_{12}	o_{11}	...									
o_{13}	...										
...											
...											
...	...	...					...	...	...	...	...
o_{21}	o_{20}	o_{19}	...				...	o_1	0	0	0
o_{22}	o_{21}	o_{20}	o_{19}	...			...	o_2	o_1	0	0
o_{23}	o_{22}	o_{21}	o_{20}	o_{19}	...	...	...	...	o_2	o_1	0
o_{24}	o_{23}	o_{22}	o_{21}	o_{20}	o_{19}	o_{18}	...	...	o_3	o_2	o_1
0	o_{24}	o_{23}	o_{22}	o_{21}	o_{20}	o_{19}	...	...	o_4	o_3	o_2
0	0	o_{24}	o_{23}	o_{22}	o_{21}	o_{20}	...	...	o_5	o_4	o_3
0	0	0	o_{24}	o_{23}	o_{22}	o_{21}	...	...	o_6	o_5	o_4

Table 5. Typical Output From a Watershed Trial

MONTHLY RAINFALL VALUES IN INCHES

DECE	JANU	FEBR	MARC	APRI	MAY	JUNE	JULY	AUGU	SEPT	OCTO	NOVE
2.230	1.230	3.630	4.500	2.530	3.550	3.270	4.190	4.060	0.770	1.430	2.420
0.800	1.470	5.030	3.240	4.040	1.960	2.150	6.070	1.430	5.800	1.990	1.370
2.900	3.390	4.670	2.080	7.230	3.560	5.760	1.270	2.420	4.970	2.780	5.900
3.410	2.070	3.650	2.750	3.460	3.320	2.390	5.470	1.910	4.450	2.570	2.500
3.920	2.840	1.780	3.460	4.290	2.950	1.970	3.570	1.780	3.810	4.250	1.610
2.370	2.860	4.170	3.080	2.450	2.260	2.570	5.590	4.640	2.890	2.200	0.810
1.430	1.370	4.500	3.160	3.110	1.990	5.650	4.140	7.800	0.880	2.540	5.450
6.330	4.510	4.170	3.490	3.590	1.050	4.500	5.360	2.410	3.350	2.630	2.620
2.870											

PRECIPITATION EXCESS

DECE	JANU	FEBR	MARC	APRI	MAY	JUNE	JULY	AUGU	SEPT	OCTO	NOVE
0.2230	0.2674	0.9502	1.4141	0.8444	0.9943	0.7821	0.6481	0.3170	0.0003	0.0000	0.0000
0.0000	0.0000	0.9803	0.8179	1.1809	1.1540	0.6210	1.3149	0.0000	1.2631	0.1305	0.0518
0.9520	3.3900	4.6700	1.1200	7.2300	0.9271	2.0959	0.0000	0.0000	0.0000	0.0800	3.7945
1.8929	0.0000	0.7750	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2.8717	2.6569	0.5346	2.0310	3.2016	2.3104	1.9700	3.0299	1.7800	3.3980	4.1895	1.6100
2.3700	2.8600	4.1700	3.0800	1.7256	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.9043	0.0000	0.0176	0.0000	1.1171	1.7679	7.5928	0.8800	2.5480	5.4500
6.3300	4.5100	4.1700	3.4900	3.5900	1.0500	4.5000	5.1811	2.4100	2.0348	0.3960	0.0000
0.0000											

MONTHLY LOSSES

DECE	JANU	FEBR	MARC	APRI	MAY	JUNE	JULY	AUGU	SEPT	OCTO	NOVE
2.0070	0.9626	2.6798	3.0859	1.6856	2.5557	2.4879	3.5419	3.7430	0.7697	1.4300	2.4200
0.8000	1.4700	4.0497	2.4221	2.8591	0.8051	1.5290	4.7551	1.4300	4.5369	1.8515	1.1182
1.9480	0.0000	0.0000	0.9592	0.0000	2.6329	3.6641	1.2700	2.4200	4.9700	2.7800	2.1055
1.5171	2.0700	2.8750	2.7500	3.4600	3.3200	2.3900	5.4700	1.9100	4.4500	2.5700	2.5000
1.0483	0.1831	1.2454	1.4290	1.0882	0.6396	0.0000	0.5401	0.0000	0.4120	0.0605	0.0000
0.0000	0.0000	0.0000	0.0000	0.7244	2.2600	2.5700	5.5900	4.6400	2.8900	2.2000	0.8100
1.4300	1.3700	3.5957	3.1600	3.0924	1.9900	4.5329	2.3721	0.2072	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1789	0.0000	1.3152	2.2340	2.4200
2.8700											

AVERAGE MONTHLY LOSSES TIMES NUMBER OF YEARS

DECE	JANU	FEBR	MARC	APRI	MAY	JUNE	JULY	AUGU	SEPT	OCTO	NOVE
8.75047	6.05579	14.44552	13.80633	12.90972	14.20330	17.17391	23.71806	14.35017	19.34369	13.12600	11.57373

RECESSION ORDINATES--ANGLE POINTS AT 1-2-4-8-16

0.28709	0.05086	0.03626	0.02167	0.00940	0.00000	0.00000	0.00000	0.00000	0.00041	0.01462	0.02862
0.04263	0.05664	0.07065	0.08465	0.07407	0.06349	0.05291	0.04233	0.03174	0.02116	0.01058	0.00000

DELTA STREAMFLOW OR SFHAT-SF

DECE	JANU	FEBR	MARC	APRI	MAY	JUNE	JULY	AUGU	SEPT	OCTO	NOVE
-0.0351	0.0066	-0.1833	-0.3275	-0.0098	0.0337	0.0960	-0.0791	0.0279	0.0953	0.0691	0.0467
0.0420	0.0369	-0.1527	-0.0758	-0.2122	0.1738	0.1823	-0.0658	0.0925	0.0164	0.0117	0.0070
0.0050	-0.0683	-0.0202	-0.0039	-0.1985	-0.0310	-0.0215	0.0599	0.0840	0.0742	0.1087	0.0418
0.0268	0.0640	0.0061	0.2148	0.3482	0.2049	0.5176	0.4515	0.4997	0.3295	0.3410	0.2477
0.1175	0.1085	0.1233	0.1592	0.2037	0.2593	0.2303	0.3526	0.1971	0.4038	0.4375	0.1643
0.2670	0.3129	0.0991	0.2109	0.3082	0.3428	0.4869	0.4172	0.4598	0.6675	0.7016	0.7538
0.7569	0.7718	0.5293	0.5116	0.4694	0.5332	0.4953	0.5294	0.5847	0.2109	0.3598	0.3381
-0.5448	-0.1803	-0.3035	-0.2072	-0.2291	-0.0213	0.5507	0.7200	0.5855	0.6974	0.6380	0.7076
0.6526											

ERROR = HR

DECE	JANU	FEBR	MARC	APRI	MAY	JUNE	JULY	AUGU	SEPT	OCTO	NOVE
0.00000	0.00000	0.00000	0.00000	-0.00001	-0.00001	-0.00001	0.00001	0.00003	0.00004	0.00006	0.00007
0.00009	0.00007	0.00005	0.00003	0.00001	-0.00001	-0.00001	-0.00001	0.32229	-0.00002	-0.00002	-0.00002
-0.00002	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00000	0.00000	0.00000	0.00000	0.00002
0.00003	0.00000	0.00004	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
-0.00003	-0.00005	-0.00006	-0.00006	-0.00007	-0.00008	0.00000	-0.00004	0.00000	-0.00001	-0.00001	0.00000
0.00000	0.00000	0.00000	0.00000	0.00009	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	-0.00006	0.00000	-0.00010	0.00000	-0.00015	-0.00020	-0.00024	0.00000	0.00000	0.00000
0.00008	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00004	0.00000	0.00009	0.00011	0.00000
0.00000											

PRECIP EXCESS / LOSS RATIO

DECE	JANU	FEBR	MARC	APRI	MAY	JUNE	JULY	AUGU	SEPT	OCTO	NOVE
0.111	0.278	0.355	0.458	0.501	0.389	0.314	0.183	0.085	0.000	0.000	0.000
0.800	0.000	0.242	0.338	0.413	1.434	0.406	0.277	0.000	0.278	0.075	0.046
0.489	999.999	999.999	1.168	999.999	0.352	0.572	0.000	0.000	0.000	0.000	1.802
1.248	0.000	0.270	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.739	14.507	0.429	1.421	2.942	3.613	999.999	5.610	999.999	8.249	69.225	999.999
999.999	999.999	999.999	999.999	2.382	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.252	0.000	0.008	0.009	0.246	0.745	36.653	999.999	999.999	999.999
999.999	999.999	999.999	999.999	999.999	999.999	999.999	26.968	999.999	1.547	0.177	0.000
0.000											

LOSS / RAINFALL RATIO

DECE	JANU	FEBR	MARC	APRI	MAY	JUNE	JULY	AUGU	SEPT	OCTO	NOVE
0.900	0.783	0.738	0.686	0.666	0.720	0.761	0.845	0.922	1.000	1.000	1.000
1.000	1.000	0.805	0.786	0.708	0.411	0.711	0.783	1.000	0.782	0.930	0.956
0.672	0.000	0.000	0.461	0.000	0.740	0.636	1.000	1.000	1.000	1.000	0.357
0.445	1.000	0.788	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.267	0.064	0.700	0.413	0.254	0.217	0.000	0.151	0.000	0.108	0.014	0.000
0.000	0.000	0.000	0.000	0.296	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.000	1.000	0.799	1.000	0.994	1.000	0.802	0.573	0.027	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.031	0.000	0.393	0.849	1.000
1.000											

APPENDIX B.

GEORGIA TECH REEC B-5500 ALGOL COMPILER   SATURDAY, 8/12/67, 5:25 AM.

```
        BEGIN

COMMENT         PROGRAM FOR SOLUTION OF LOSSES WITH WATER YIELD MODEL;
FILE IN         CRDRBT(2,10);
FILE OUT        RYTRBT 6(2,15);
INTEGER         I,J,K,L,M,N,MO,X,W,PP;
REAL            INTI,INTII,INTIII,INTIV,INTV,INTVI,E,PCOR;
ARRAY           RF[1:300],S[1:300],MOFCT[1:300],P[1:300],SF[1:300],
                AA[1:5,1:300],ATXA[1:5,1:5],ATXSF[1:5],
                ATXAINV[1:5,1:5],DVALUE[1:5],SFHAT[1:300],DELTSF[1:300],
                D[1:24],DM[1:27,1:27],DD[1:27,1:11],DTXD[1:11,1:11],
                DTXDINV[1:11,1:11],DELSHAT[1:300],DTXDLSF[1:27],
                RR[1:300],ERR[1:300],LOSS[1:300],LOSSAVG[1:12],
                LOSSRF[1:300],PELOSS[1:300];
LABEL           DOOVER,EXIT,A300,A400,RITE,LAST;
FORMAT          FMT1("LOOP NUMBER  ",I2,/),

                FMT2("MONTHLY RAINFALL VALUES IN INCHES",/),
                FMT3(12F10.3),
                FMT4("PRECIPITATION EXCESS",/),
                FMT5(12F10.4),
                FMT6("MONTHLY LOSSES",/),
                FMT7("AVERAGE MONTHLY LOSSES TIMES NUMBER OF YEARS",/),
                FMT8(12F10.5),
                FMT9("RECESSION ORDINATES--ANGLE POINTS AT 1-2-4-8-16"),
                FMT10("DELTA STREAMFLOW OR SFHAT-SF",/),
                FMT11("ERROR- - RR",/),
                FMT12(12F6.2),
                FMT13("   DECE        JANU        FEBR        MARC        APRI
  MAY        JUNE       JULY        AUGU        SEPT        OCTO        NOVE",/),
                FMT14("                                              ILLINOIS WATE
RSHED - BUREAU CREEK -101 SQ MI",/),
                FMT15("PRECIP EXCESS / LOSS   RATIO",/),
                FMT16("LOSS / RAINFALL  RATIO",/),
                FMT20("DIVISION BY ZERO"),
                FMT21("OUT OF DATA");

LIST            LST1(FOR J+24 STEP 1 UNTIL MO DO RF[J]),
                LST2(FOR J+24 STEP 1 UNTIL MO DO P[J]),
                LST3(FOR J+1 STEP 1 UNTIL MO-23 DO LOSS[J]),
                LST4(FOR J+1 STEP 1 UNTIL 12 DO LOSSAVG[J]),
                LST5(FOR J+1 STEP 1 UNTIL 24 DO D[J]),
                LST6(FOR K+1 STEP 1 UNTIL MO-23 DO DELTSF[K]),
                LST7(FOR J+1 STEP 1 UNTIL MO-23 DO RR[J]),
                LST8(FOR J+1 STEP 1 UNTIL MO-23 DO PELOSS[J]),
                LST9(FOR J+1 STEP 1 UNTIL MO-23 DO LOSSRF[J]);
PROCEDURE       INVERT(N,A,A200);
VALUE           N;
INTEGER         N;
ARRAY           A[1,1];
LABEL           A200;
        BEGIN
INTEGER         I,J,K;

                FOR K+1 STEP 1 UNTIL N DO
        BEGIN
                IF A[K,K]=0 THEN GO TO A200;
                FOR J+1 STEP 1 UNTIL K-1,K+1 STEP 1 UNTIL N DO
                A[K,J]+A[K,J]/A[K,K];
                A[K,K]+1/A[K,K];
```

```
            FOR I←1 STEP 1 UNTIL K-1,K+1 STEP 1 UNTIL N DO
        BEGIN
            FOR J←1 STEP 1 UNTIL K-1,K+1 STEP 1 UNTIL N DO
            A[I,J]←A[I,J]-A[I,K]×A[K,J];
            A[I,K]←-A[I,K]×A[K,K];
        END;
        END;
        END INVERT;

COMMENT     NEXT CARD IS MONTH CARD;
            MO←288;
            READ(CRDRRT,FMT12,FOR K←1 STEP 1 UNTIL MO DO RF[K],
                          FOR K←1 STEP 1 UNTIL MO DO  S[K],
                          FOR K←1 STEP 1 UNTIL 12 DO MOFCT[K])[EXIT];
            CLOSE(CRDRRT,RELEASE);
            N←1;
            FOR K←1 STEP 1 UNTIL MO DO
        BEGIN
            P[K]←RF[K]-MOFCT[N];
            N←N+1;
            IF N>12 THEN N←1
        END;
COMMENT     FORM P MATRIX NOTE THAT SF[1] = RO[24];
COMMENT     ANGLE POINTS IN RECESSION  = 1,2,4,8,16,24;
DOOVER:     FOR I←1 STEP 1 UNTIL (MO-23) DO
        BEGIN
            N←I;
            SF[I]←S[I+23]-P[N+23];
            AA[1,I]←P[N+22]+(1/2)×P[N+21]-(3/2)×P[N+23];
            AA[2,I]←(1/2)×P[N+21]+P[N+20]+(3/4)×P[N+19]+(1/2)×P[N+18]
                    +(1/4)×P[N+17]-3×P[N+23];
            AA[3,I]←(1/4)×P[N+19]+(1/2)×P[N+18]+(3/4)×P[N+17]+
                    P[N+16]+(7/8)×P[N+15]+(3/4)×P[N+14]+(5/8)×P[N+13]
                    +(1/2)×P[N+12]+(3/8)×P[N+11]+(1/4)×P[N+10]
                    +(1/8)×P[N+9]-6×P[N+23];
            AA[4,I]←(1/8)×P[N+15]+(1/4)×P[N+14]+(3/8)×P[N+13]+
                    (1/2)×P[N+12]+(5/8)×P[N+11]+(3/4)×P[N+10]+
                    (7/8)×P[N+9]+P[N+8]+(7/8)×P[N+7]+(3/4)×P[N+6]+
                    (5/8)×P[N+5]+(1/2)×P[N+4]+(3/8)×P[N+3]+
                    (1/4)×P[N+2]+(1/8)×P[N+1]-8×P[N+23]
        END;
COMMENT     MULTIPLICATION OF ATRANSPOSED BY MATRIX AA;
            FOR J←1 STEP 1 UNTIL 4 DO
            FOR K←1 STEP 1 UNTIL 4 DO
        BEGIN
            INTI←0.0;
            FOR I←1 STEP 1 UNTIL MO-23 DO
            INTI←AA[K,I]×AA[J,I]+INTI;
            ATXA[K,J]←INTI
        END;
COMMENT     MULTIPLICATION OF ATRANSPOSED BY SF VECTOR;
            FOR K←1 STEP 1 UNTIL 4 DO
        BEGIN
            INTII←0.0;
            FOR I←1 STEP 1 UNTIL MO-23 DO
            INTII←AA[K,I]×SF[I]+INTII;
            ATXSF[K]←INTII
        END;
COMMENT     PROCEDURE TO INVERT ATXA MATRIX;
COMMENT     (NOTE THAT W=N,ATXAINV=A,A300=A200);
            FOR J←1 STEP 1 UNTIL 4 DO
            FOR L←1 STEP 1 UNTIL 4 DO
```

```
                    ATXAINV[L,J]+ATXA[L,J];
                    W+4;
                    INVERT(W,ATXAINV,A300) ;
COMMENT             MULTIPLICATION OF ATXAINV BY ATXSF;
                    FOR L+1 STEP 1 UNTIL 4 DO
               BEGIN
                    INTIII+0.0;
                    FOR J+1 STEP 1 UNTIL 4 DO
                    INTIII+ATXAINV[L,J]×ATXSF[J]+INTIII;
                    OVALUE[L]+INTIII
               END;
COMMENT             EVALUATE SFHAT FROM OVALUES =(AAMATRIX × OVALUES);
                    FOR I+1 STEP 1 UNTIL (MO-23) DO
               BEGIN
                    INTIV+0.0;
                    FOR J+1 STEP 1 UNTIL 4 DO
                    INTIV+AA[J,I]×OVALUE[J];
                    SFHAT[I]+INTIV
               END;
COMMENT             FORMATION OF [SFHAT-SF] VECTOR;
COMMENT             DELTA SF[J] = SFHAT-SF(OBSERVED);
                    FOR J+1 STEP 1 UNTIL (MO-23) DO
                    DELTSF[J]+SFHAT[J]-SF[J];
COMMENT             FORMATION OF OMATRIX FOR BACKWARD SOLUTION;
                    O[1]+1.0-1.5×OVALUE[1]-3.0×OVALUE[2]-6.0×OVALUE[3]
                         -8.0×OVALUE[4];
                    O[2]+OVALUE[1];
                    O[3]+0.5×OVALUE[1]+0.5×OVALUE[2];
                    O[4]+OVALUE[2];
                    O[5]+0.75×OVALUE[2]+0.25×OVALUE[3];
                    O[6]+0.5×OVALUE[2]+0.5×OVALUE[3];
                    O[7]+0.25×OVALUE[2]+0.75×OVALUE[3];
                    O[8]+OVALUE[3];
                    FOR J+1 STEP 1 UNTIL 7 DO
                    O[8+J]+((8-J)/8)×OVALUE[3]+(J/8)×OVALUE[4];
                    FOR J+0 STEP 1 UNTIL 8 DO
                    O[16+J]+((8-J)/8)×OVALUE[4];
                    FOR I+1 STEP 1 UNTIL 24 DO
               BEGIN
                    IF O[I]>1.0 THEN O[I]+1.0;
                    IF O[I]<0.0 THEN O[I]+0.0
               END;
                    INTVI+0.0;
                    FOR I+1 STEP 1 UNTIL 24 DO
                    INTVI+INTVI+O[I];
                    FOR I+1 STEP 1 UNTIL 24 DO
                    O[I]+O[I]/INTVI;
                    FOR I+1 STEP 1 UNTIL 27 DO
                    FOR J+1 STEP 1 UNTIL 27 DO
                    IF J>I THEN OM[I,J]+0.0 ELSE
                    IF I-J≥24 THEN OM[I,J]+0.0 ELSE
                    OM[I,J]+O[I-J+1];
COMMENT             FORMATION OF OO MATRIX;
                    FOR I+1 STEP 1 UNTIL 27 DO
               BEGIN
                    OO[I,1]+(1/6)×(OM[I,2]+OM[I,12])+(1/3)×(OM[I,3]+OM[I,11])
                           +(1/2)×(OM[I,4]+OM[I,10])+(2/3)×(OM[I,5]+OM[I,9])
                           +(5/6)×(OM[I,6]+OM[I,8])+OM[I,7];
                    OO[I,2]+(1/6)×OM[I,8]+(1/3)×OM[I,9]+(1/2)×OM[I,10]
                           +(2/3)×OM[I,11]+(5/6)×OM[I,12]+OM[I,13]+(4/5)
                           ×OM[I,14]+(3/5)×OM[I,15]+(2/5)×OM[I,16]+(1/5)
                           ×OM[I,17];
```

```
                OD[I,3]←(1/5)×OM[I,14]+(2/5)×OM[I,15]+(3/5)×OM[I,16]
                      +(4/5)×OM[I,17]+OM[I,18]+(1/2)×OM[I,19];
                OD[I,4]←(1/2)×OM[I,19]+OM[I,20];
                FOR J←5 STEP 1 UNTIL 11 DO OD[I,J]←OM[I,J+16]
           END;
COMMENT         OD TRANSPOSED TIMES OD;
                FOR J←1 STEP 1 UNTIL 11 DO
                FOR K←1 STEP 1 UNTIL 11 DO
           BEGIN
                INTVI←0.0;
                FOR I←1 STEP 1 UNTIL 27 DO
                INTVI←INTVI+OD[I,K]×OD[I,J];
                OTXD[K,J]←INTVI
           END;
COMMENT         INVERT DTXD;
                FOR I←1 STEP 1 UNTIL 11 DO
                FOR J←1 STEP 1 UNTIL 11 DO
                DTXDINV[I,J] ← DTXD[I,J];
                X←11;
                INVERT(X,DTXDINV,A400);
COMMENT         OD TRANSPOSED TIMES DELTSF;
                FOR I←1 STEP 1 UNTIL 11 DO
           BEGIN
                INTV←0.0;
                FOR J←1 STEP 1 UNTIL 27 DO
                INTV←INTV+OD[J,I]×DELTSF[J];
                DTXDLSF[I]←INTV
           END;
COMMENT         SOLUTION FOR FIRST 23 ERRORS;
                FOR K←1 STEP 1 UNTIL 7 DO
           BEGIN
                E←0.0;
                FOR J←1 STEP 1 UNTIL 11 DO
                E←E + DTXDINV[K,J]×DTXDLSF[J];
                ERR[K]←E
           END;
                RR[1]←0.0;
                FOR L←2 STEP 1 UNTIL 7 DO
                RR[L]←(ERR[1]×(L-1))/6;
                FOR L←8 STEP 1 UNTIL 12 DO
                RR[L]←ERR[1]×((13-L)/6)+ERR[2]×((L-7)/6);
                RR[13]←ERR[2];
                FOR L←14 STEP 1 UNTIL 17 DO
                RR[L]←ERR[2]×((18-L)/5)+ERR[3]×((L-13)/5);
                RR[18]←ERR[3];
                RR[19]←0.5×ERR[3]+0.5×ERR[4];
                FOR L←20 STEP 1 UNTIL 23 DO RR[L]←ERR[L-16];
                FOR I←1 STEP 1 UNTIL 23 DO
           BEGIN
                PCOR←P[I+23]-RR[I];
                IF PCOR>RF[I+23] THEN
           BEGIN
                RR[I]←P[I+23]-PCOR;
                P[I+23]←RF[I+23]
           END
                ELSE IF PCOR<0.0 THEN
           BEGIN
                RR[I]←P[I+23]-PCOR;
                P[I+23]←0.0
           END
                ELSE P[I+23]←PCOR
           END;
```

```
COMMENT        COMPUTATIONS FOR THE MIDDLE MONTHS 24-(MO-3);
               FOR L←24 STEP 1 UNTIL MO-26 DO
          BEGIN
COMMENT        DTXDLSF SEQUENTIALLY;
               FOR I←1 STEP 1 UNTIL 11 DO
          BEGIN
               INTV←0.0;
               FOR J←1 STEP 1 UNTIL 27 DO
               INTV←INTV+DD[J,I]×DELTSF[J+L-24];
               DTXDLSF[I]←INTV
          END;
COMMENT        SOLUTION FOR MIDDLE ERRORS;
               E←0.0;
               FOR J←1 STEP 1 UNTIL 11 DO
               E←E + DTXDINV[8,J]×DTXDLSF[J];
               PCOR←P[L+23]-E;
               IF PCOR>RF[L+23] THEN
          BEGIN
               RR[L]←P[L+23]-RF[L+23];
               P[L+23]←RF[L+23]
          END
               ELSE IF PCOR<0.0 THEN
          BEGIN
               RR[L]←P[L+23];
               P[L+23]←0.0
          END
               ELSE
          BEGIN
               RR[L]←E;
               P[L+23]←PCOR
          END;
          END;
COMMENT        CORRECT FORWARD THREE MONTHS;
               FOR M←L+1 STEP 1 UNTIL L+3 DO
               DELTSF[M]←DELTSF[M]-RR[L]×D[M-L+1];
COMMENT        LAST 3 MONTHS ERROR;
               FOR L←MO-25 STEP 1 UNTIL MO-23 DO
          BEGIN
               E←0.0;
               FOR J←1 STEP 1 UNTIL 11 DO
               E←E+DTXDINV[L-MO+34,J]×DTXDLSF[J];
               RR[L]←E;
               PCOR←P[L+23]-RR[L];
               IF PCOR>RF[L+23] THEN
          BEGIN
               RR[L]←P[L+23]-RF[L+23];
               P[L+23]←RF[L+23]
          END
               ELSE IF PCOR<0.0 THEN
          BEGIN
               RR[L]←P[L+23];
               P[L+23]←0.0
          END
               ELSE P[L+23]←PCOR;
          END;
               PP←PP+1;
               WRITE(RYTRBT,FMT1,PP);
               FOR I←24 STEP 1 UNTIL MO-23 DO
          BEGIN
               IF ABS(RR[I])>0.05 THEN GO TO DOOVER
          END;
               FOR J←1 STEP 1 UNTIL MO-23 DO
```

```
             LnSS[J]+RF[J+23]=P[J+23];
             FnR K+1 STEP 1 UNTIL 12 DO
        BEGIN
             InTV+0.0;
COMMENT      LnSSAVG TIMES MONTHS IN COLUMN OR CUMULATIVE LOSS;
             FnR J+K STEP 12 UNTIL MO=34+K DO
        BEGIN
             InTV+INTV+LOSS[J];
             LnSSAVG[K]+INTV
        END;
             FnR J+1 STEP 1 UNTIL MO=23 DO
             IF RF[J]=0.0 THEN LOSSRF[J]+999.999 ELSE
             LnSSRF[J]+LOSS[J]/RF[J+23];
             FnR J+1 STEP 1 UNTIL MO=23 DO
             IF LOSS[J]=0.0 THEN PELOSS[J]+999.999 ELSE
             PELOSS[J]+P[J+23]/LOSS[J];
        END;
RITE:   BEGIN            REAL DUM;

             WRITE(RYTRBT,FMT2);
             WRITE(RYTRBT,FMT13);
             WRITE(RYTRBT,FMT3,LST1);
             WRITE(RYTRBT,FMT4);
             WRITE(RYTRBT,FMT13);
             WRITE(RYTRBT,FMT5,LST2);
             WRITE(RYTRBT,FMT6);
             WRITE(RYTRBT,FMT13);
             WRITE(RYTRBT,FMT5,LST3);
             WRITE(RYTRBT,FMT7);
             WRITE(RYTRBT,FMT13);
             WRITE(RYTRBT,FMT8,LST4);
             WRITE(RYTRBT,FMT9);
             WRITE(RYTRBT,FMT8,LST5);
             WRITE(RYTRBT,FMT10);
             WRITE(RYTRBT,FMT13);
             WRITE(RYTRBT,FMT5,LST6);
             WRITE(RYTRBT,FMT11);
             WRITE(RYTRBT,FMT13);
             WRITE(RYTRBT,FMT8,LST7);
             WRITE(RYTRBT,FMT15);
             WRITE(RYTRBT,FMT13);
             WRITE(RYTRBT,FMT3,LST8);
             WRITE(RYTRBT,FMT16);
             WRITE(RYTRBT,FMT13);
             WRITE(RYTRBT,FMT3,LST9);
         END;

             GO TO LAST;
 A300:       WRITE(RYTRBT,FMT20);
             GO TO LAST;
 A400:       WRITE(RYTRBT,FMT20);
             GO TO LAST;
 EXIT:       WRITE(RYTRBT,FMT21);
 LAST:       END.
```

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